Chaos-assisted tunnelling with cold atoms

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I — What is chaos-assisted tunnelling ?

II — Theoretical and experimental model with cold atoms.

III — How to characterize chaos-assisted tunnelling ?

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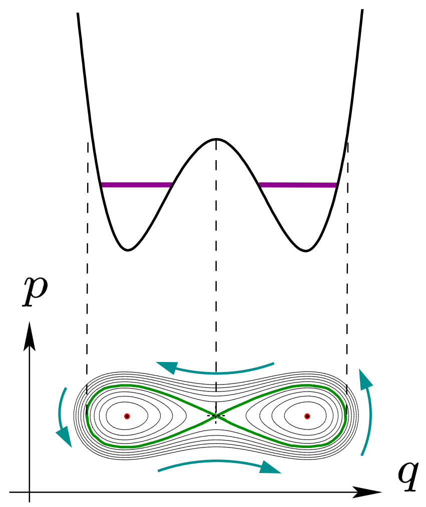
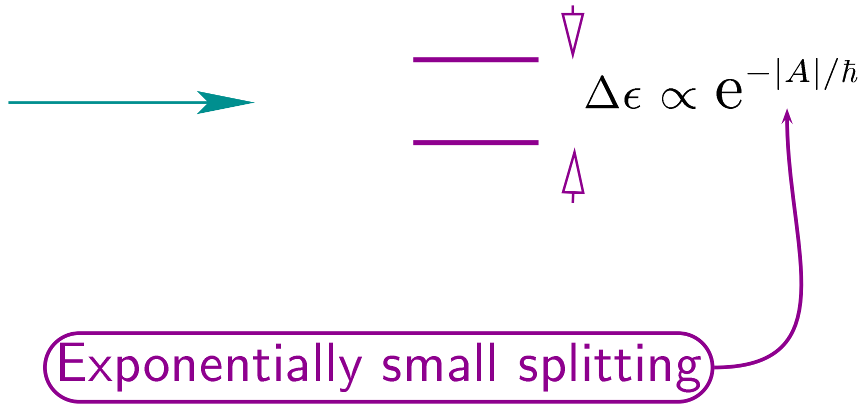
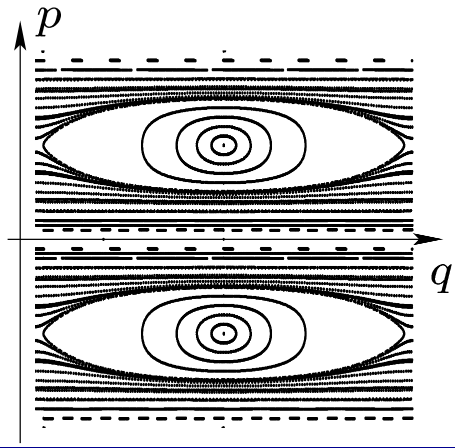
Tunnelling and chaos

Regular double well





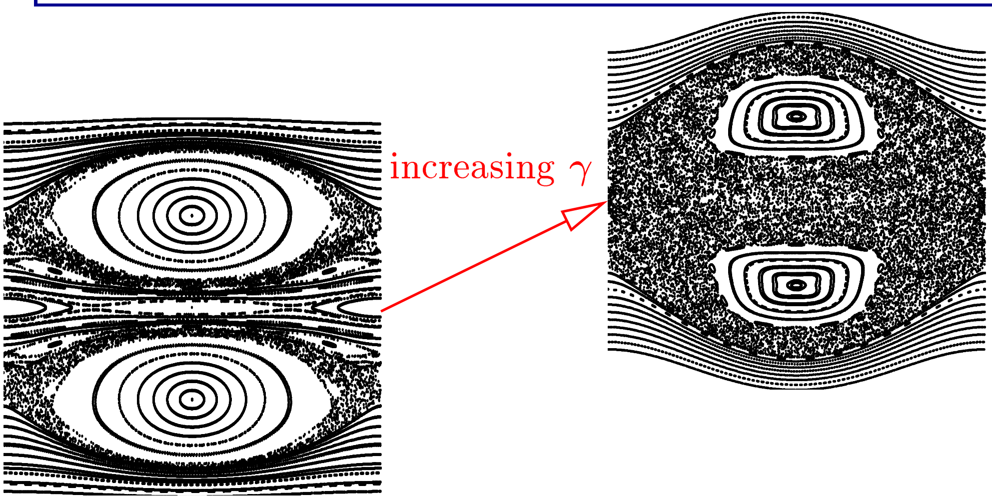
Generalization 1: dynamical tunnelling



discrete symmetry

(here: $p \longrightarrow -p$)

Generalization 2: regular dynamical tunnelling + γ chaos



Quantum chaos question: What is the signature of (classical) chaos on the (quantum) $\Delta\epsilon$'s ?

What is chaos-assisted tunnelling ?

☞ First general answer:
Between two stable symmetric islands, chaos erases the classical barrier and hence the *average* tunnelling effect is exponentially enhanced.

☞ Refinements:
When a quantum or a classical parameter is varied, the *fluctuations* of $\Delta\epsilon$ should be different in the chaotic case compared to the regular case.

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☞ Motivation:
This work lies in the area of *transport and wave propagation in complex media*.

Cold atoms and tunnelling

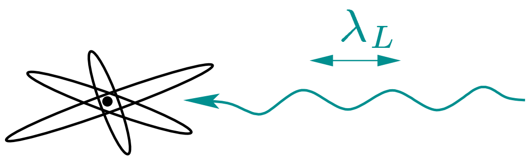
rich well-understood internal structure (alkali)

no Coulomb forces ($\neq$ ions or electrons)

Cold neutral independent atoms interacting with lasers

to preserve quantum coherence and to prevent dissipation

few degrees of freedom ($\neq$ BEC)



dipole forces: $-\overrightarrow{\mathrm{grad}}(\vec{p}.\vec{E}+\vec{\mu}.\vec{B})$

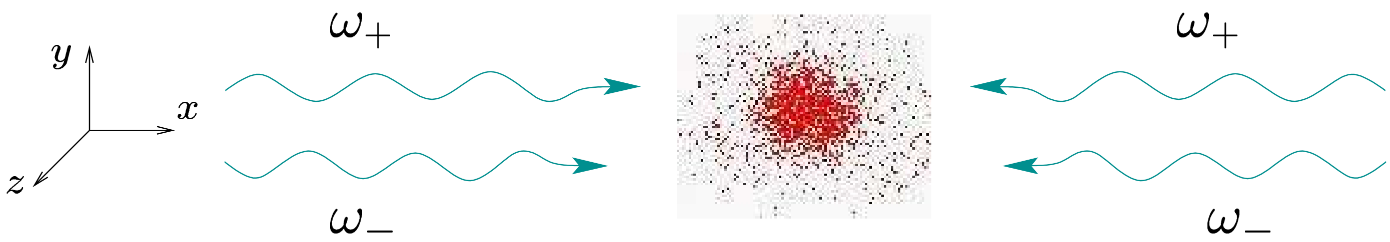
$\Longrightarrow$ Quantum system with few degrees of freedom governed by an effective Hamiltonian

Tunnelling:

- A. Mouchet *et al.*, Phys. Rev. E (2001)
- W. K. Hensinger *et al.*, Nature (2001)
- D. A. Steck *et al.*, Science (2001), Phys. Rev. Lett. (2002)

Experimental setup

The atoms evolve in two counter-propagating far-detuned standing waves with slightly different frequencies $\omega_\pm = \omega_L \pm \delta\omega/2$ with $\delta\omega \ll \omega_L$.



Each atom sees an effective time-dependent, 1D, hamiltonian:

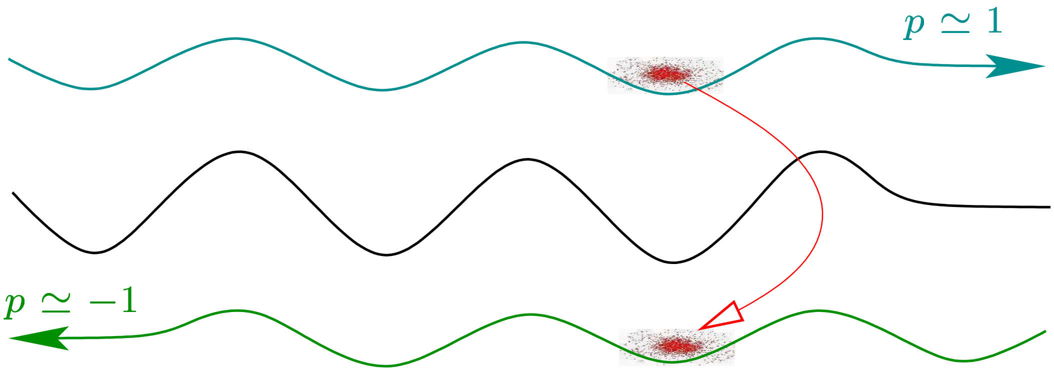
$$H = \frac{p_x^2}{2M} - V_0 \cos(2k_L x)[\theta + \cos(\delta\omega\, t)]$$

in reduced variables:

$$\begin{aligned} H_{\text{eff}}(p,q;\tau) &= H_0(p) + \gamma H_1(p,q;\tau) \\ &= \frac{1}{2}p^2 - \gamma\,\theta\cos(q) - \frac{\gamma}{2}\cos(q+\tau) - \frac{\gamma}{2}\cos(q-\tau). \end{aligned}$$

with $[q,p] = i\hbar_{\text{eff}}$ where $\underbrace{\hbar_{\text{eff}} = 8\omega_R/\delta\omega}$ and $\omega_R = \hbar k_L^2/2M$.

can be experimentally tuned



Experimental signal

Displaying the momentum distribution of the atomic cloud leads to the measurement of the tunnelling period

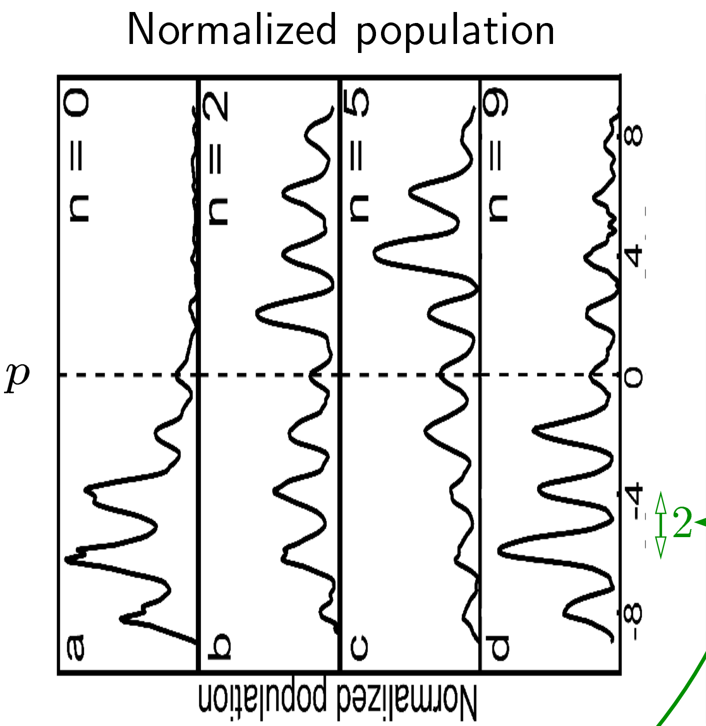
$$T_{\text{tunnel}} \simeq \frac{\hbar_{\text{eff}}}{\Delta\epsilon}$$

Orders of magnitude:

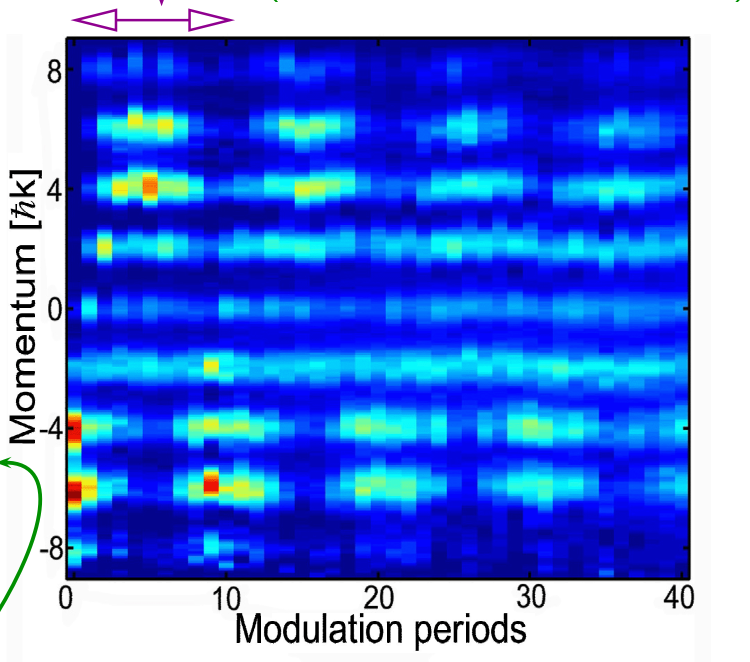
Na (Hensinger *et al.*): $\delta\omega/2\pi \sim 250$ kHz, $\gamma \sim 1.0$, $\theta \sim 1.7$, $\hbar_{\text{eff}} \sim 0.8$;

Cs (Steck *et al.*): $\delta\omega/2\pi \sim 300$ kHz, $\gamma \sim 0.3$, $\theta \sim 1.0$, $\hbar_{\text{eff}} \sim 0.3$,

$$T_{\text{tunnel}} \sim 200\ \mu s \sim 10 \times 2\pi/\delta\omega$$



(©ourtesy: W. K. Hensinger)



effect of the spatial periodicity (coherence over one Brillouin zone)

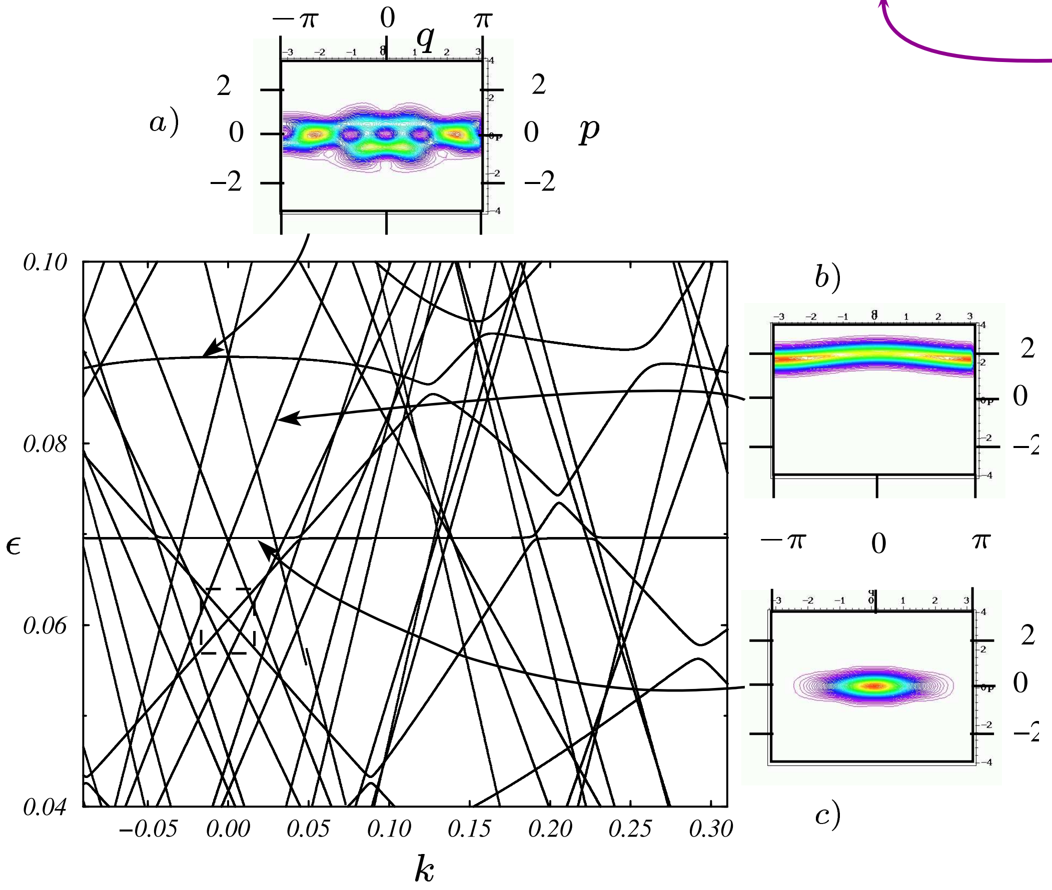
Quantum spectra

Quasi-energies for a q,τ-periodic Hamiltonian:

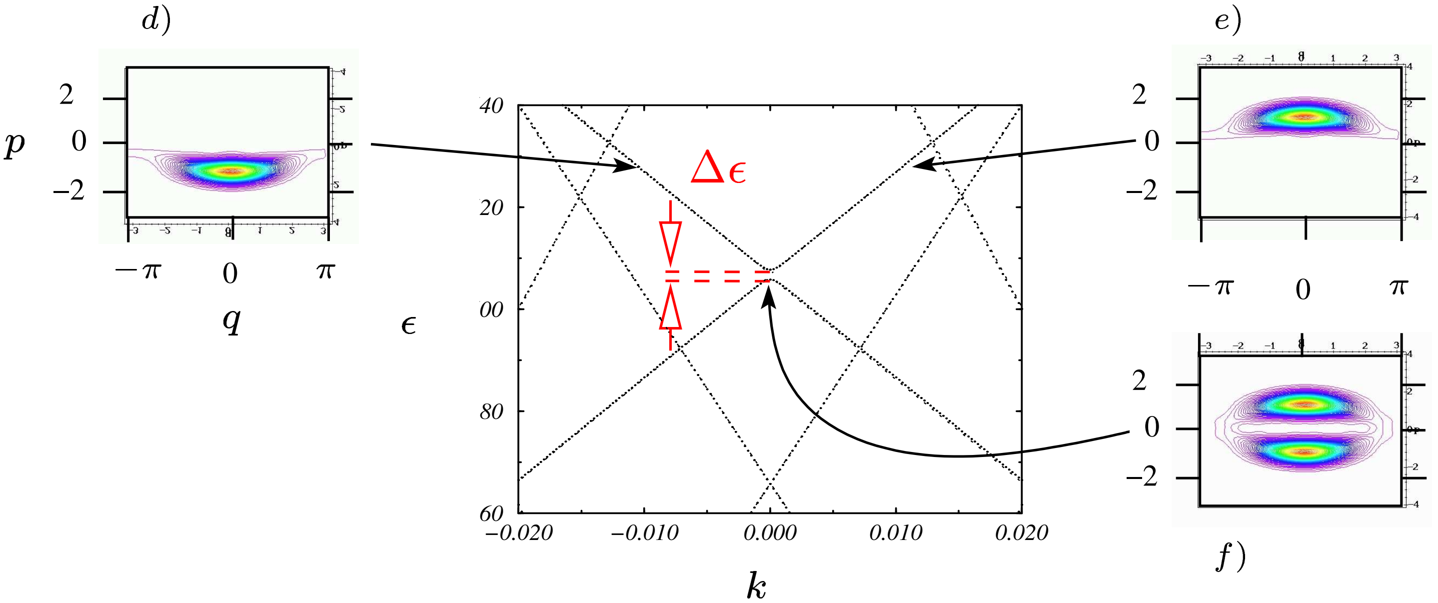
$$U(t{+}\tau,t)\,|\,\psi_\epsilon(t)\,\rangle = \mathrm{e}^{-\mathrm{i}\epsilon\tau/\hbar}|\,\psi_\epsilon(t)\,\rangle \Rightarrow \left|\begin{array}{l}\text{bands } \epsilon_n(k)\\ k\in \text{1st Brillouin zone}\end{array}\right.$$

Husimi representation: $\psi^{\mathrm{H}}(p,q) \propto |\langle\, z\,|\,\psi\,\rangle|^2$

$z{=}p{+}\mathrm{i}q$

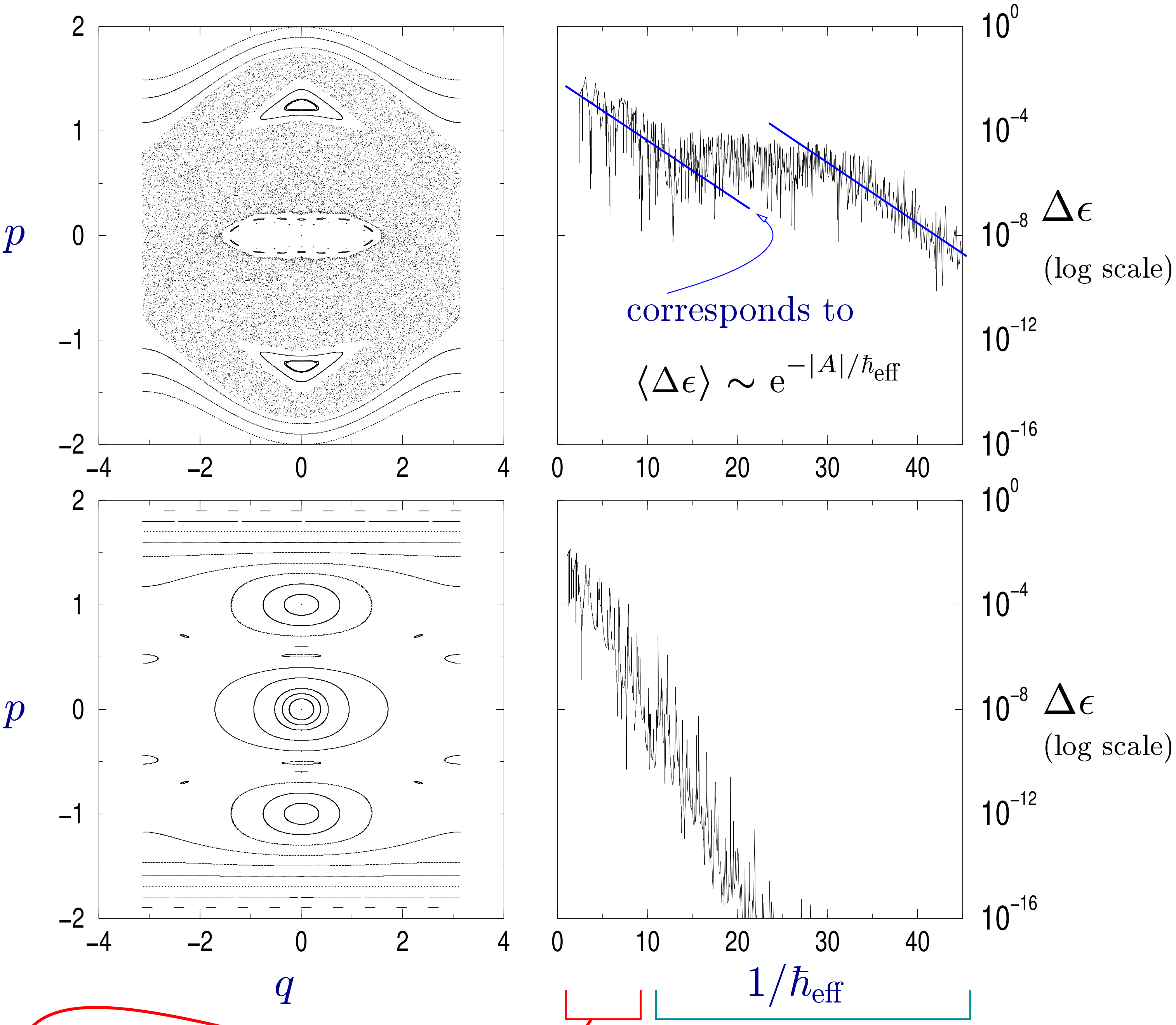


$\gamma = .18$
$\hbar_{\mathrm{eff}} = .2037$



How to characterize chaos-assisted tunnelling ?

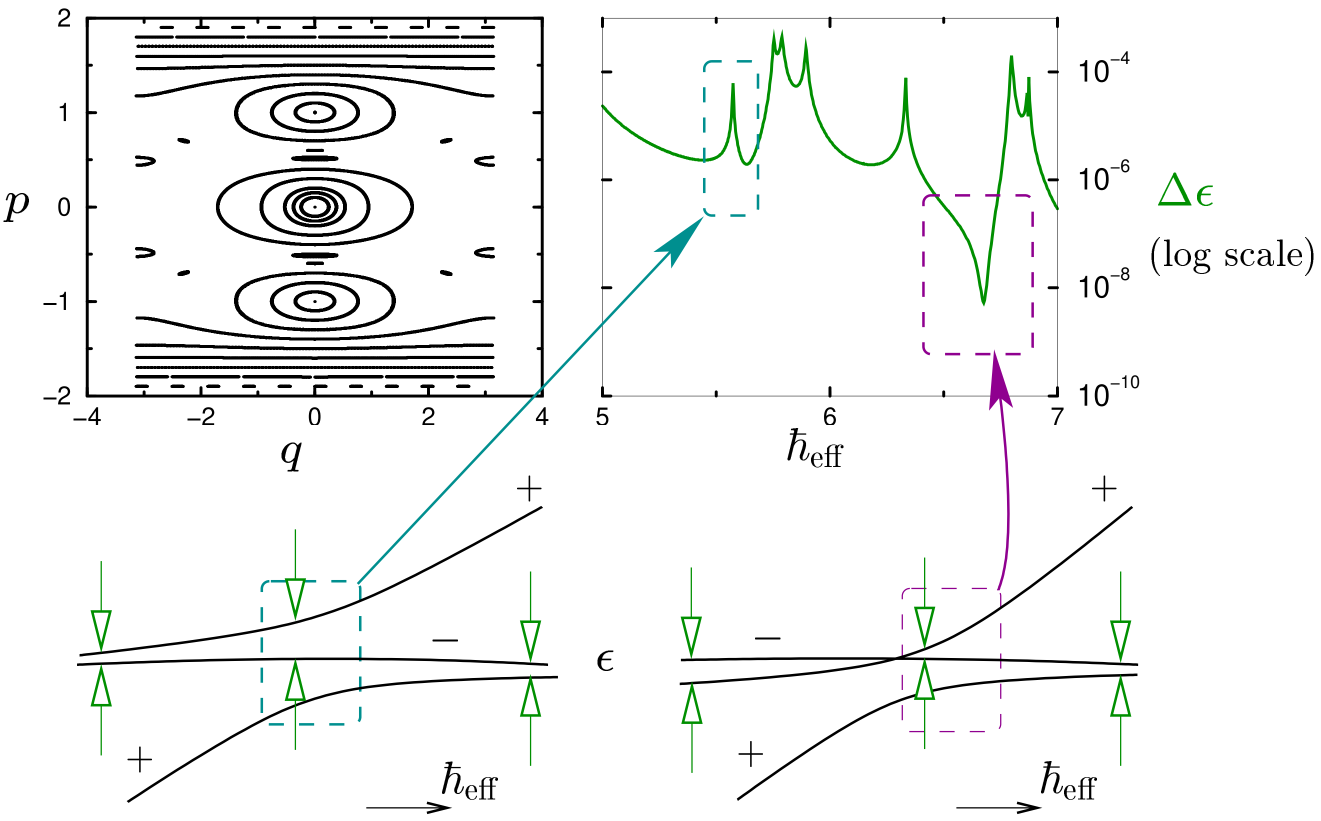
1. Effect on the average splitting.



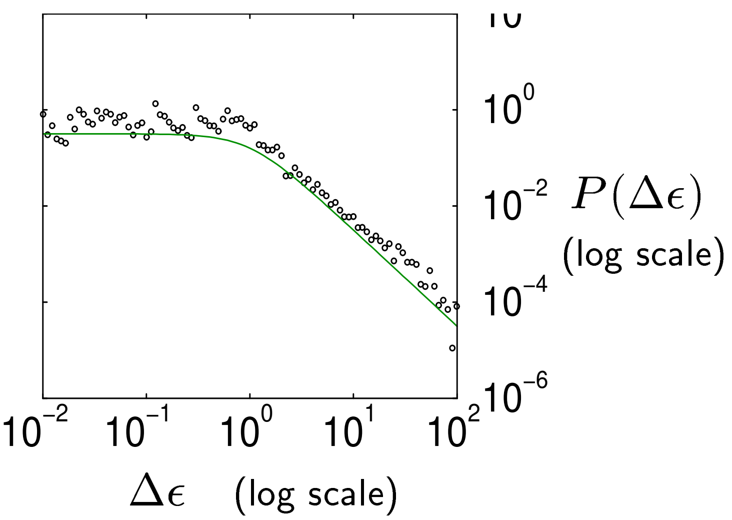
If $\hbar_{\text{eff}}$ is too small the averages $\langle \Delta\epsilon \rangle_{\text{regular}}$ and $\langle \Delta\epsilon \rangle_{\text{chaotic}}$ cannot be distinguished because of the huge fluctuations.

Discussion of the proposed signatures of chaos-assisted tunnelling

2. Is a huge variation a signature ? No !



3. Is LEYVRAZ-ULLMO law a signature ? No !!



$$P(\Delta\epsilon) = \frac{1}{\pi}\frac{1}{1+\Delta\epsilon^2}.$$

Random matrix theory with a random gaussian coupling between the tunnelling doublet and the other states.

Perspectives

☞ Theoretical:

- Finding unambiguous signatures of chaos-assisted tunnelling;
- Better understanding of Leyvraz-Ullmo law;

☞ Experimental:

- Decreasing $\hbar_{\mathrm{eff}}$ by one order of magnitude: $\delta\omega/2\pi \sim 1$ MHz.

References:

Tunnelling in $d > 1$ dimensions:

- M. J. Davis and E. J. Heller, J. Phys. Chem. (1981);
- M. Wilkinson, Physica D (1986);
- W. A. Lin and L. E. Ballentine, Phys. Rev. Lett. (1990);
- O. Bohigas, D. Boosé, R. Egydio de Carvalho, and V. Marvulle, Nuclear Phys. A (1993).

Chaos-assisted tunnelling:

- O. Bohigas, S. Tomsovic & D. Ullmo, Phys. Rep. (1993);
- F. Leyvraz and D. Ullmo, J. Phys. A (1996).

See also:

- S. C. Creagh and N. D. Whelan, Phys. Rev. Lett. (1996);
- O. Brodier, P. Schlagheck & D. Ullmo, Phys. Rev. Lett. (2001).