QUANTUM COMPUTATION AND SPIN ELECTRONICS

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Abstract. [1] In this chapter we explore the connection between mesoscopic physics and quantum computing. After giving a bibliography providing a general introduction to the subject of quantum information processing, we review the various approaches that are being considered for the experimental implementation of quantum computing and quantum communication in atomic physics, quantum optics, nuclear magnetic resonance, superconductivity, and, especially, normal-electron solid state physics. We discuss five criteria for the realization of a quantum computer and consider the implications that these criteria have for quantum computation using the spin states of single-electron quantum dots. Finally, we consider the transport of quantum information via the motion of individual electrons in mesoscopic structures; specific transport and noise measurements in coupled quantum dot geometries for detecting and characterizing electron-state entanglement are analyzed.

1. Brief Survey of the History of Quantum Computing

The story of why quantum computing and quantum communication are theoretically interesting and important has been told in innumerable places before, and we will just point the reader to some of those. The "prehistory" of quantum computing (up to 1994) consists of the tinkerings of a small number of visionaries on the question of how data could be processed if bits could be put into quantum superpositions of states. The idea of a quantum

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gate was introduced, the basic possibilities of quantum algorithms were set forth, quantum communication (in the form of quantum cryptography) was well developed, and some rudimentary ideas of how physical systems could be made to implement quantum computing were considered. Actually, it is humbling to reflect on how few people it took (no more than about 15, we would say) to launch a set of basic concepts which have now come to occupy the attention of some hundreds of researchers.

Here is a bibliography of recent and not-so-recent review articles which would bring the reader up to speed on quantum computation and related areas. Those interested in seeing how complete the "prehistoric" perspective was should consult Ekert's influential paper at Atomic Physics 14 [1], which helped to make atomic physicists the earliest participants in the attempt to build a quantum computer. Another early but influential review which is a very good source for the Shor algorithm is [2]. An early general overview is [3], and an early simple discussion of quantum gate constructions is [4]. More recent general overviews are [5] and [6]. [7] and [8] give a set of ideas for the future direction of quantum communication and information theory. Many surveys have considered the question of how quantum computers might be implemented in actual physical devices; for the solid-state type of implementations which we will discuss below, we might mention [9] besides some of the other articles already cited.

2. Creating the Quantum Computer

Of course, the reason for this contribution to appear in a school on mesoscopic quantum phenomena is that we hope, ultimately, that out of mesoscopic physics will emerge the capability actually to build a working, scalable quantum computer. Microelectronic mesoscopic devices will inevitably emerge, of course, but whether they will be usable for quantum computation will depend on whether they manage to satisfy a very specific set of requirements.

2.1. FIVE CRITERIA FOR BUILDING A QUANTUM COMPUTER

These requirements can be boiled down to a list of five criteria. We have written frequently about these five criteria before[8, 9], but they are worth considering in detail here again, since we have found that these criteria provide a unifying framework for these investigations which encompass an astonishingly broad and rich range of fundamental physics. Further investigations have endowed this simple list of five with more and more richness and interest, and show how exciting the building of a quantum computer will be from the point of view of novel and basic physics.

Before getting into the serious work, we find it amusing to state the criteria stripped of all physics language and posed purely as a set of requirements for building a computer. In this light our five points are extremely trivial:

1. The machine should have a collection of bits.
2. It should be possible to set all the memory bits to 0 before the start of each computation.
3. The error rate should be sufficiently low.
4. It must be possible to perform elementary logic operations between pairs of bits.
5. Reliable output of the final result should be possible.

It has to be admitted that no great brainwork is required to arrive at this set of basic attributes which a computer ought to possess. But let us take the next step and translate these into a physics language, to specify what it really takes to achieve these very basic and simple requirements in a quantum setting:

1. A physical system with a collection of well characterized quantum two-level systems (qubits) is needed. Each qubit should be separately identifiable and externally addressable. The dynamics of the system should be under sufficient control that these qubits are never excited into any third level. It should be possible to add qubits at will.
2. It should be possible to, with high accuracy, completely decouple the qubits from one another, and it should be possible to start an experiment by placing each qubit in its lower (0) state.
3. The decoherence time of these qubits should be long, ultimately up to 10^4 times longer than the "clock time" (see the next requirement).
4. Logic operations should be doable. This involves having the one-body Hamiltonian of each qubit under independent and precise control (this gives the one-bit gates). Two-body Hamiltonians involving nearby qubits should also be capable of being turned on and off under external control (these are the two-bit gates). In a typical operation, the one- or two-body Hamiltonian will be turned on smoothly from zero to some value and then turned off again, all within one clock cycle; the integral of this pulse should be controllable to again about one part in 10^4.
5. Projective quantum measurements on the qubits must be doable. It is useful, but not absolutely necessary, for these measurements to be doable fast, within a few clock cycles. It is also useful, but not necessary, for the measurement to have high quantum efficiency (say 50%). If the quantum efficiency is many orders of magnitude lower, then the quantum computation must be done in an "ensemble" style, in which many identical quantum computers are running simultaneously. As

with all the other requirements, these measurements absolutely must be qubit specific.

The reader will notice that there is a lot more to be said about these criteria as physics requirements than as computer requirements. Indeed, we will see in the examples we review below that these criteria involve not just innovations in materials preparation (for numbers 1 and 3), not just novel device design and fabrication (for number 4), not just new, precision high-speed electronics at the nanometer scale (number 4), not just unprecedented capabilities for ultrasensitive metrology (number 5), but all of these at once!

2.2. POTENTIAL REALIZATIONS

There is a remarkably long list of physical systems that have been proposed, and are under active experimental investigation, for the creation of a quantum computer. This list is remarkable not only for its length but also for its diversity; it seems that virtually every area of quantum physics has a candidate. (This is not quite true: we know of no proposed quantum computer emerging from high-energy particle physics.)

Since non-solid state implementations are not the focus of this article, we will merely mention the many approaches in this category and give a bibliography: In atomic physics, the original proposal by Cirac and Zoller[10] considered the internal states of ions as the qubits, coupled by the common vibrational mode of the ions in the trap. Later this group[11] introduced a variant in which the coupling is provided by a common cavity electromagnetic mode. This work has inspired several analogous solid state proposals which we will mention below. Many other variants of the cavity quantum electrodynamics schemes have been proposed; in some of these the qubit could be transmitted from one subprocessor to another as a single photon (in an optical fiber, say)[12]; these schemes are intriguing in that they offer an idea of how to do distributed quantum computing and distributed decision problems.

A very recent development from atomic physics, neutral-atom optical lattices, has been proposed to provide the necessary tools to build a quantum computer[13]. Here, the ability to move trapped atoms in a state-dependent way by changing the (classical) phase of the trapping laser beams is exploited to obtain the necessary conditional dynamics to do two-bit gates. There are many ideas for mixing and matching these various atomic-physics proposals.

Another large area of research has involved bulk NMR of small molecules containing sets of spin-1/2 nuclei in solution[14, 15]. This represents an "ensemble" approach, exploiting the ability to read out the result of quantum computation with only low quantum efficiency measurements (criterion 5 above). Unfortunately these schemes presently do not satisfy criterion 2 above, but in other respects it represents a possible approach to a scalable quantum computer.

An "almost" solid state approach, which we will mention just because it illustrates that quantum computing proposals are coming from just about every area of physics these days, is one proposed by Platzman and coworkers[16] involving two dimensional crystalline layers of electrons that can be trapped near the surface of liquid He. We are not knowledgeable enough to comment on this proposal in any detail (it seems to involve elements of the quantum dot proposals we will discuss in a moment, plus some ideas from the atomic physics proposals), but it is interesting to see contact made with another, evidently quite well developed, area of research in quantum physics.

3. Solid State Proposals

There have been almost as many proposals for solid state implementations of quantum computers as all the other proposals put together. We think there are some clear reasons for this: we believe that solid state physics is the most versatile branch of physics, in that almost any phenomenon possible in physics can be embodied in a correctly designed condensed matter system. Even rather esoteric properties of field theories which are of interest in high-energy physics have useful realizations, for example, in the fractional quantum Hall effect. A related reason is that solid state physics, being so closely allied with computer technology, has exhibited great versatility over the years in the creation of artificial structures and devices which are tailored to show a great variety of desired physical effects. This has been exploited very powerfully to produce ever more capable computational devices. It would be natural to extrapolate to say that this versatility will extend to the creation of solid state quantum computers as well; the plethora of proposals would indicate that this is indeed true, although time will tell whether any of these proposals will actually provide a successful route to a quantum computer!

3.1. THE SPINTRONIC QUANTUM DOT PROPOSAL

We will first go through, in some detail, the proposals that we[17] have made for the use of coupled quantum dot arrays for quantum computation; we will also discuss several closely related proposals such as the "Kane model"[18, 19]. Our proposal may rightly be termed a "spintronic" proposal, as it attempts to use as much as possible the tools of standard electronics (control via voltage gates, readout by current detection) to accomplish quantum computing; but our proposal departs from conventional

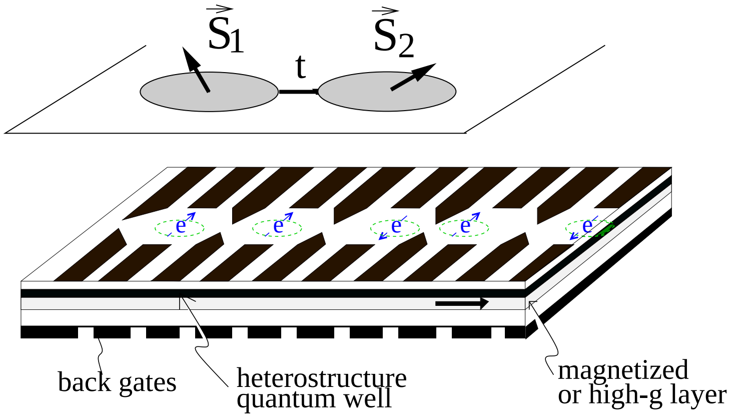

Figure 1. Above: Two coupled quantum dots, between which electrons can tunnel with amplitude t. This tunneling leads to an effective spin interaction $J \sim t^2/U$ (where U is the on-site Coulomb repulsion) between the excess spins $\mathbf{S}_1$ and $\mathbf{S}_2$ in the dots, which can be controlled by a number of external parameters. In principle, any type of tunnel-coupled confined structure is a candidate for the "spintronics" quantum computing proposal. E.g., the dots can be defined electrically in a two-dimensional electron gas (as suggested by this drawing), or they can be vertically coupled dots, or even atoms, *etc.* Below: Concept for quantum-dot array device.

electronics in that it uses the *spin* of the electrons as the basic information carrier.

We think that **Criterion 1** above is most naturally satisfied by a genuine spin-1/2 system, which by its nature has a doubly-degenerate ground state to serve as a qubit. As we will discuss, the prospects for long coherence times are better for this qubit than for many other "pseudo-spin" degrees of freedom that might be chosen. (But we will mention several other possible ones being considered at the end of this section.)

We have to specify how one obtains the individually identifiable and addressable qubits of Criterion 1. Our scheme for doing this involves an extension of well-demonstrated techniques in the area of mesoscopic quantum-dot physics. In particular, we are interested in quantum dots created by lateral confinement in a two-dimensional electron gas. The structure is illustrated in Fig. 1. To have qubits in this structure (one per quantum dot) using the electron spin, it is necessary for the electron number to be controlled precisely; this is accomplished using the well-documented Coulomb blockade effect. The electron number must be fixed at an odd value in order for the spin quantum number to be 1/2. This can be achieved if the electron number can be controllably tuned down to one (technically difficult in structures in which the Coulomb blockade is probed by transport measurements, quite feasible in structures where the Coulomb blockade is sensed capacitively); alternatively, a consensus seems to be developing that a dot with a spin quantum number of 1/2 gives a characteristic zero-bias anomaly in transport which signals a Kondo effect. This approach will have to be used with caution, however–the spin quantum number of a dot may readily switch from 1/2 to 3/2 as a function of the dot shape[20]; it is of course essential that this switching does not occur between the configuration in which a qubit is characterized and that in which it is used.

Criterion 1 also requires that the next higher lying state should never (or hardly ever) be occupied; this requires that dots be sufficiently small that the next level is inaccessible to thermal excitation or non-adiabatic excitation. It appears that this requirement will be quite readily satisfied for dots of the 10s of nanometer sizes that are already studied, at normal (l-He) cryogenic temperatures. As for extendibility, as our Figure 1 suggests this is "simply" a matter of making an array of dots, either one or two dimensional. Of course, this cartoon does not do justice to the actual complexity of the multilevel wiring layout required to address a large array of qubits; we hope we are justified in considering this an "engineering" matter, although by no means a trivial one.

In the other spin-qubit scheme we have considered[19], the "quantum dot" is provided by a single donor impurity atom lying near the surface of a Si-Ge heterostructure. Such a system is a kind of natural one-electron quantum dot; the hydrogenic impurity potential can normally only bind a single conduction-band electron (sometimes it can bind two: see Criterion 5 below). The characteristics of this qubit (its g-factor, orbital size, and so forth) are more nearly unique than in the lateral quantum dot proposal; this could ultimately be either an a advantage or a disadvantage. Also, placing individual impurities at specified locations near the surface of a semiconductor is a very daunting technical challenge, although perhaps one that the technological world will be needing to face in any case in the coming nanoelectronic age. In general, Criterion 1 poses very serious challenges in the art of material preparation and device fabrication, as our discussion should make clear.

Criterion 2 is a relatively straightforward one, but involves additional device-fabrication considerations. In order for the quantum-dot or donor-impurity qubit to start in a stable all-0 state, it is sufficient to place the spins in a several-Tesla magnetic field at liquid-He temperatures; then the probability of occupying the all-down ground state is sufficiently high. In order for this state to be stable upon removal of the magnetic field, we require a probably more difficult requirement: the qubits must be, to rather high accuracy (to be discussed further in the next two criteria), decoupled from one another. The device geometry is chosen so that, in the "resting" state, the "point contact" voltage probes (the pairs of electrodes between neighboring quantum dots) can be set to a high repulsive voltage so that the overlap between neighboring quantum-dot orbitals is negligible. Turning this voltage to low can increase this overlap by orders of magnitude, and this is an essential computational step (see Criterion 4).

In the donor-impurity scheme, top electrodes control the near-surface band bending of the semiconductor; in the flat-band condition, the donor hydrogenic orbitals are compact enough that the overlap between neighbors is negligible. By suitable band bending, the electron orbitals can be made much more delocalized, leading to orders of magnitude more spin-spin interaction.

Criterion 3 probably involves the must fundamental quantum physics, in that it relates to the issues of the decoherence of quantum systems and the transition between quantum and classical behavior. Of course, a lot of attention has been devoted in fundamental mesoscopics research to characterizing and understanding the decoherence of electrons in small structures. We remind the reader, however, that most of what has been probed (say in weak localization studies or the Aharonov-Bohm effect) is the *orbital* coherence of electron states, that is, the preservation of the relative phase of superpositions of spatial states of the electron (e.g., in the upper or lower arm of an Aharonov-Bohm ring). The coherence times seen in these investigations are almost completely irrelevant to the *spin* coherence times which are important in our quantum computer proposal. There is some relation between the two if there are strong spin orbit effects, but our intention is that conditions and materials should be chosen such that these effects are weak.

Under these circumstances the spin coherence times (the time over which the phase of a superposition of spin-up and spin-down states is preserved) can be completely different from the charge coherence times, and in fact it is known that they can be orders of magnitude longer. This was actually one of our prime motivations for proposing spin rather than charge as the qubit in these structures. The experimental measurement of this kind of coherence is not so familiar in mesoscopic physics, but fortunately it is very familiar in the area of spectroscopy. The measurement to probe spin coherence is essentially equivalent of the characterization of the so-called T_2 time in spin resonance.

In fact, we suggest that the quantification of spin coherence in the quantum dot quantum computer will follow a line very familiar from the many recent experiments of Awschalom, Kikkawa, and collaborators [21] that have observed spin precession in a variety of semiconductor materials. We expect that these experiments will set up a large array of identical quantum dots; the dots don't have to be fully "wired up" for quantum computation, but they should otherwise have the same device structure as in the quantum computer (because we expect that the coherence times should be very device- and structure-specific). The spins will be set in a superposition of up and down in a magnetic field, and the decay of free induction as these spins precess in the field will be observed. Conventional "spin echo" tricks should be used to eliminate the effects of residual inhomogeneities. Note that this measurement only requires low quantum efficiency, so it is not nearly so difficult as the quantum measurements which we will discuss in Criterion 5.

Awschalom and coworkers have already done many such measurements on a variety of semiconductor systems, and sees decoherence times up to hundreds of nanoseconds in some structures[21]. For donor impurities in silicon, traditional electron spin resonance measurements have seen T_2 times for the P donor spin up to hundreds of microseconds. As we see decoherence times in the range from microseconds to milliseconds as acceptable for quantum computation, these results are very encouraging; but they are not conclusive. Decoherence times, depending on the details of the quantum degrees of freedom in the environment that the qubit can become entangled with, are expected to be very sensitive to the details of the makeup of the physical device. Since none of the experiments have been done on an actual quantum computing structure as we envision it, the existing results cannot be viewed as conclusive. Because of this sensitivity to details, theory can only give general guidance about the mechanisms and dependencies to be looked for, but cannot make reliable *a priori* predictions of the decoherence times.

In fact there are further complications in store: we know theoretically that decoherence is not actually fully characterized by a single rate; in fact, a whole set of numbers is needed to fully characterize the decoherence process (12 in principle for individual qubits), and no experiment has been set up yet to completely measure this space of parameters, although the theory of these measurements is available. Even worse, decoherence effects will in principle be modified by the act of performing quantum computation (during gate operation, decoherence is occurring in a coupled qubit system[17]). We believe that the full characterization of decoherence will involve ongoing iteration between theory and experiment, and will probably be inseparable from the act of building a reliable quantum computer.

Finally, the decoherence time τ_ϕ by itself is not a figure of merit of a quantum computer proposal — the amount of coherent computation which can be performed depends on the ratio τ_ϕ/τ_s where τ_s denotes the switching time (τ_s^{-1} is the "clock frequency" of the quantum computer).

Criterion 4 also requires an extensive discussion of the physics of the proposed qubit device. It is necessary to identify simple, reliable mechanisms by which specified qubits can be subjected to one-body (one-bit gates) and two-body (two-bit gates) Hamiltonians which can be turned on and off in time. Almost all the complexity of the proposed devices arises from the need to achieve these capabilities.

In the quantum dot array structure, the two-bit gates are obtained by a controlled lowering of the potential barrier produced by the "point contact" gates between neighboring quantum dots. When this barrier is lowered, the two electrons are brought together, forming, temporarily, an artificial hydrogen molecule. The effective spin-spin interaction that this produces should, to high accuracy, be given by a Heisenberg interaction $J\vec{S}_i \cdot \vec{S}_{i+1}$, where the exchange coupling should be tunable up to about 0.1 meV [22], with the "off" value being many orders of magnitude lower than this. The exponentially strong suppression of the coupling J in the "off" state of the gate is essential, because it assures that no correlated errors occur due to the switching mechanism. Corrections to the Heisenberg form should arise only from relativistic effects (spin-orbit coupling); although spin-orbit corrections to the band parameters like the g-factor are fairly large in GaAs, the residual relativistic effects on the low-angular momentum effective-mass conduction band states can be estimated to be negligibly small [9](c).

For laterally tunnel-coupled quantum dots in a two-dimensional electron system it was found that besides electrical gating, an external magnetic field can be used to switch on and off the spin-spin interaction [22]. Recently, there has been great interest in vertically tunnel-coupled dot structures, both in etched vertical columnar heterostructures[23] and in double-layer self-assembled quantum-dot structures[24]. We have analyzed the spin-interaction in such vertically coupled quantum dots[25], and have found, in addition to the ones known from laterally coupled dots, a new mechanism which allows the external control of the spin interaction. In this scenario, two coupled quantum dots of different size are subject to an external electric field. The field shifts the big dot by a larger distance than the small dot, therefore leading to an effective increase in the inter-dot distance, which causes an exponential suppression of the spin exchange coupling.

The proposed two-bit gate mechanism in the donor-impurity quantum computer is conceptually almost identical to the proposals above. By weakening the binding of the electron to the impurity by band bending, the orbitals can be made to spread out so that the overlap of neighboring orbitals becomes appreciable; the exchange physics, and the expected effective Hamiltonian, is the same as the quantum dot case.

It is known that universal, fault tolerant quantum computation can be obtained with the logic gates obtainable from this nearest neighbor exchange interaction [26]. For dots of tens of nanometer size, a smooth (i.e. adiabatic) turning on and off the Hamiltonian on a time scale of 10s or 100s of picoseconds would be desirable. (The smoothness is required so that higher-lying states of the dot are not unintentionally excited [22].)

This is technically feasible, although it requires very high-bandwidth control (10-100 GHz) of the voltages on each individual electrode (Fig. 1) of the structure, which is a daunting job of microwave engineering.

Especially daunting is the theoretical precision requirement, which is that the integrated strength of the exchange Hamiltonian should be controlled to about one part in 10^4. This number, based on the analysis of the efficacy of error correction techniques in quantum computation, may come down as better error correction schemes are devised. Also, if this number were "only" 10^2 in an experiment, many interesting studies of quantum computation could still be performed. As an example, we have described a minimal experimental test for quantum error correction involving (at least) three coupled quantum dots[27]. Time will tell what the ultimate technical requirements will be. A fortunate fact about GaAs quantum dot structures, holding out the hope that precision manipulation will be possible, is the observation that, when treated correctly, quantum dot structures show essentially no "charge switching" effects which are the origin of traditional 1/f noise in transport[28]. Note that uncontrolled charge switching is not nearly so great a problem for spin qubits as for charge qubits, since this switching does not couple directly to the spin degree of freedom. But, since the second order effects of charge motion could change the strength of the exchange coupling J by more than a part in 10^{-4}, the ability to suppress 1/f effects will be very important for switching in quantum computation.

One-qubit gates must also be considered, and these involve rather different physics. Theoretically the requirement is very simple for a spin-1/2 qubit: it must be possible to subject a specified qubit to a (real or effective) magnetic field of specified direction and strength. We have offered many suggestions previously[17, 22] on how this requirement may be met: by the application of real, localized magnetic fields using a scanned magnetic particle or nanoscale electric currents; by the use of a magnetized dot or magnetized barrier material that the electron can be inserted in and out of by electric gating; by the judicious choice of g-factor-modulated materials[8]. We have performed some detailed analysis of this last mechanism recently, and it is also the preferred mechanism in the Si-Ge heterostructure scheme (Si and Ge have very different g-factors), so we will discuss this recent work here.

Due to spin-orbit coupling, the Landé g-factor in bulk semiconductor materials differs from the free-electron value $g_0 = 2.0023$ and ranges from large negative to large positive numbers for various materials. In confined structures such as quantum wells, wires, and dots, the g-factor is modified with respect to the bulk material and sensitive to an external bias voltage[29]. Here, we study the simpler case of a layered structure in which the effective g-factor of electrons is varied by electrically shifting their equilibrium position from one layer (with g-factor g_1) to another (with another g-factor $g_2 \neq g_1$). For simplicity, we use the bulk g-factors of the layer materials, an approximation which becomes increasingly inaccurate as the layers become thinner[30].

We consider a quantum well (e.g. AlGaAs-GaAs-AlGaAs), in which some fraction y of the Ga atoms are replaced by In atoms in the upper half of the heterostructure (we have used $y = 0.1$). The sequence of layers in the heterostructure is then $Ga_{1-x}Al_xAs$-GaAs-$Ga_{1-y}In_yAs$-$Ga_{1-x-y}Al_xIn_yA$ s, where x denotes the Al content in the barriers (typically around 30%). Changing the vertical position of the electrons in the quantum well via top or back gates permits control of the effective g-factor for the corresponding electrons: If the electron is mostly in a pure GaAs environment, then its effective g-factor will be around the GaAs bulk value (g_{GaAs}=-0.44) whereas if the electron is in the InGaAs region, the g-factor will be somewhere between the GaAs and the InAs values (g_{InAs}=-15). We have analyzed the problem of a single electron in such a structure, neglecting screening due to surrounding electrons. This procedure is justified, since we are interested in isolated electrons located in quantum dots. In a quantum well with a high electron density, however, many-body effects should be taken into account.

We have solved the one-dimensional problem,

$$\left[-\frac{d}{dz}\frac{\hbar^2}{2m(z)}\frac{d}{dz}+V(z)\right]\Psi(z)=E\Psi(z), \tag{1}$$

of a single electron with a spatially varying effective mass $m(z)$ numerically by discretizing it in real space and subsequently performing exact diagonalization. The potential $V(z)$ describes the quantum well (conduction band offset $\Delta E_c = 270\,\mathrm{meV}$) and the electric field E in growth direction. For the effective masses and g-factors of the various layers we have linearly interpolated between the GaAs, AlAs, and InAs values. The resulting effective g-factor was calculated by averaging the g-factor over the electronic ground-state wavefunction,

$$g_{\mathrm{eff}} = \int dz g(z)|\Psi(z)|^2. \tag{2}$$

In Fig. 2, we plot the effective g-factor g_{eff} versus the electric field for a quantum well which is $w = 10\,\mathrm{nm}$ wide. For the barrier thickness we have assumed $w_{\mathrm{B}} = 10\,\mathrm{nm}$. At moderate electric fields, g_{eff} interpolates roughly between the GaAs and $Ga_{1-y}In_yAs$ g-factors. If the electric energy $eEw_{\mathrm{B}} = eU_{\mathrm{B}}$ becomes larger than the barrier ΔE_c, we observe a vertical deconfinement of the electrons. In our plot (Fig. 2) the electric deconfinement is clearly seen as a jump of the effective g-factor to the barrier material

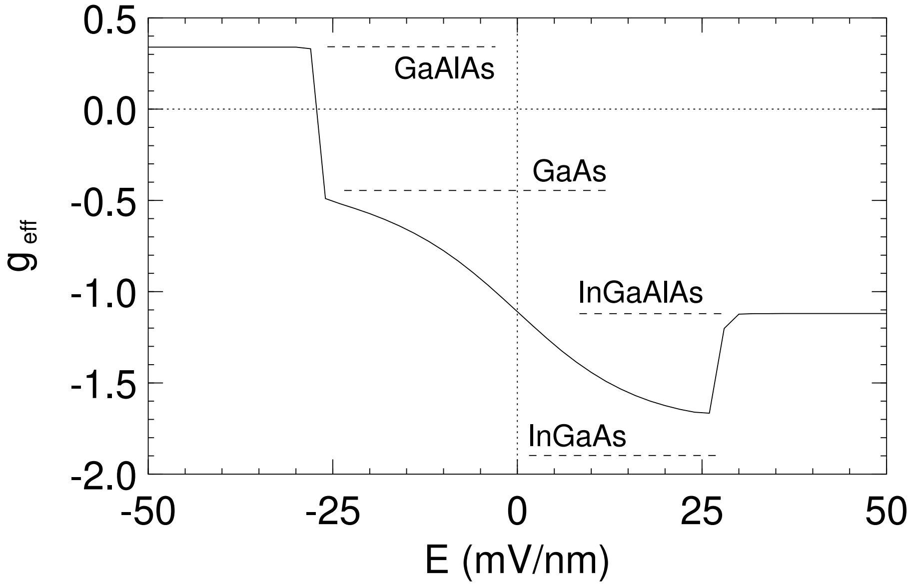

Figure 2. The effective g-factor g_{eff} of electrons confined in a $\text{Ga}_{1-x}\text{Al}_x\text{As}$ - GaAs - $\text{Ga}_{1-y}\text{In}_y\text{As}$ - $\text{Ga}_{1-x-y}\text{Al}_x\text{In}_y\text{As}$ heterostructure ($x = 0.3$, $y = 0.1$) as a function of the applied electric field E in growth direction. The widths of the quantum well and the barriers are $w = w_{\text{B}} = 10\,\text{nm}$. The g-factors which are used for the materials are indicated with horizontal lines.

value at $E = \pm\Delta E_c/ew_{\text{B}} = \pm 27\,\text{mV/nm}$. The electric field required for a substantial change in g_{eff} is of the order of $10\,\text{mV/nm}$, corresponding to a voltage of $100\,\text{mV}$, which is about one order of magnitude smaller than the band gap ($1.5\,\text{eV}$ for GaAs at $T = 0$).

Since the g-factor is spatially varying, the Zeeman coupling influences the electron wavefunction, which in principle could lead to a non-linear spin splitting $\Delta E(B)$. For the above materials and parameters however, we find numerically that the splitting is almost exactly linear, $\Delta E(B) \simeq g_{\text{eff}}\mu_B B$. The irrelevance of the Zeeman coupling for the orbital wavefunction is due to the fact that the typical electronic kinetic energy is at least two orders of magnitude larger than the typical Zeeman energy.

The described quantum well can host an array of electrostatically defined quantum dots, containing a single excess electron (and thus a single spin 1/2) each. In order to carry out a single-qubit operation on one of the spins, the whole system is placed into a homogeneous magnetic field. By changing the voltage at the electric gate on top of a single quantum dot, the effective g-factor g_{eff} for the spin in this quantum dot can be changed by about $\Delta g_{\text{eff}} \approx 1$ with respect to the g-factor of all remaining spins. This leads to a relative rotation about the direction of $\mathbf{B}$ by an angle of roughly

$\phi = \Delta g_{\text{eff}} \mu_B B \tau / 2\hbar$. The typical switching time τ for a $\phi = \pi/2$ rotation using a field of 1 T is then approximately $\tau \approx 2\phi\hbar/\Delta g_{\text{eff}}\mu_B B \approx 30$ ps. Controlling the top gate at $\tau^{-1} \approx 30$ GHz seems very challenging; we emphasize however that the single-qubit operation can be done much more slowly (a lower limit is provided by the spin dephasing time). The switching can be slowed down either by choosing a smaller Δg_{eff} or by replacing ϕ by $\phi + 2\pi n$ where n is an integer.

Actually, we should note that there is a substantial degree of flexibility in how universal quantum computation is achieved. We have noted in our original work[17] that switchable effective magnetic fields on the dots are not needed for the implementation of one bit gates, if there are some dots which have a higher static magnetic field, either because they are magnetized or because of the presence of a fixed magnetic field gradient. Then, one-qubit operations can be effected by swapping the qubit onto the magnetized dot, then swapping if off again once the desired interaction with the magnetic field has occurred.

In yet another variant along these lines, Bacon *et al.* [31] have very recently shown that, at the price of increasing the number of quantum-dot spins required to represent each qubit, all computation can be done with exchange interactions alone, without the need for any local magnetic fields, except during the measurement operation. These workers define a logical qubit as the two-level system of the singlet states of four spins, in which

$$\begin{aligned} |0_L\rangle &= |S\rangle \otimes |S\rangle, \\ |1_L\rangle &= \tfrac{1}{\sqrt{3}}[|T_+\rangle \otimes |T_-\rangle - |T_0\rangle \otimes |T_0\rangle + |T_-\rangle \otimes |T_+\rangle]. \end{aligned} \tag{3}$$

Here $|S\rangle$ is the singlet state of two spins and $|T_{+,-,0}\rangle$ are the three triplet states of two spins. The initial preparation of $|0_L\rangle$ is easy; introduce a strong exchange interaction between pairs of spins (e.g., on the "even" bonds but not on the "odd" bonds of a one-dimensional chain); the ground state of this Hamiltonian is the desired state. Ref. [31] shows that all necessary one- and two-qubit gates between these logical qubits can be done by sequences of exchange interactions only. (More work should be done to make these sequences explicit and short.) This is possible because the exchange interactions can move the quantum state vector anywhere within the total singlet (total spin=0) sector. Ref. [31] also notes that the measurement of the logical qubit (anticipating Criterion 5 below) can be done if the first two spins in the four-spin block are measured in the z bases and the next two are measured in the x basis.

We have recently noted that even this potentially inconvenient requirement for two different measurement bases can be eliminated. Only z measurements are needed, if it is assumed that other spins, initialized in the $|0\rangle$ state (*not* $|0_L\rangle$), are also available, which is possible if some spins are

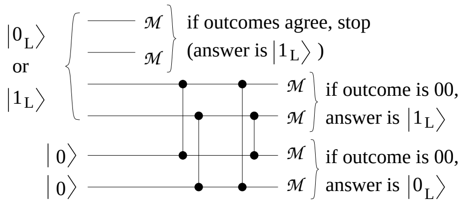

Figure 3. Two ancilla spins set to the $|0\rangle$ state suffice to measure the logical qubit $|0_L, 1_L\rangle$. First, the first two spins of the logical qubit are measured in the z basis. If the two measurement outcomes agree, then the procedure is finished, the outcome is $|1_L\rangle$. Otherwise, the ancilla must be used; first, the four two-bit gates are performed in the sequence indicated. The gate is the square-root-of-swap; see [17, 27]. After this circuit, all four remaining spins are measured in the z basis. If the top two measurements give 00, the outcome is $|1_L\rangle$; if the bottom two measurements give 00, the outcome is $|0_L\rangle$. No other measurement results are possible.

initially cooled in a uniform external field without being exchange-coupled to their neighbors. Then, the measurement protocol of Fig. 3 suffices.

This little digression about coded qubits illustrates a more general theme here: the theory indicates that many tradeoffs are possible, in this case between the number of spins needed and the complexity of the required gate operations. Each specific experimental case needs to be carefully assessed to see what tradeoffs are possible, and how they can be optimized.

We think that **Criterion 5** hold the prospect for some early successes in this long road we have laid out to achieving a quantum computer, successes that will advance some fundamentally interesting agendas in solid state physics. This criterion can be simply posed: how do we measure the state of the spin of a single electron in a solid? Actually there is a lot of flexibility in this requirement, in that high quantum efficiency is not in principle needed: if the quantum efficiency is 1%, say, then the computer can still give a reliable output if the qubit to be read is first copied, say, 200 times, then each of these qubits is subjected to this low-efficiency measurement. (Note that this does not violate the no-cloning theorem, because the extra qubits act as "copies" only in the basis specified for the measurement). But there are various reasons why high quantum efficiency is desirable: obviously, the number of extra qubits needed is less, but also, it is desirable for error correction (although not absolutely necessary) that the whole measurement be complete, and the measurement outcome available, in a time much shorter (by a factor of 100, say) than the decoherence time.

Indeed, we think that there is a real prospect that such fast, high-sensitivity measurements can be achieved for spins. This is so despite the fact that direct, magnetometric measurements of the spin via its magnetic moment have, after many years of effort[32], so far failed to achieve anything close to single-spin sensitivity; it is for this reason that we are pessimistic about a magnetic-force-microscope based quantum computer[33]. We feel that a very promising general strategy involves first converting the spin degree of freedom of the electron, which is very hard to measure, into a charge degree of freedom, which is measurable at the single-electron level by well-established electrometric techniques. There are several possible ways that this could be achieved; we have previously discussed[17, 34] a scheme in which a partially transparent barrier (say, the bottom wall of the quantum dot) has a barrier height which is spin-dependent, either because of a large g-factor or an exchange splitting of the barrier. Then, the measurement would consist of applying a bias voltage to the dot so that the electron is pressed up against this barrier; if the spin is up, say, the barrier height is low, so the electron has a high chance of tunneling and being detected by an electrometer circuit underneath; while if the spin is down, the barrier height is high, tunneling does not take place, and the electron is not detected.

In the donor-impurity scheme we can use a similar approach, which uses some interesting physics of the donor impurity that was pointed out by Kane[18]. The phosphorus impurity in Si has a stable singly charged state (two electron state), but only if the relative spin of the two electrons is a singlet. If we wish to measure whether the spins on two neighboring Ps are in a singlet or triplet (not identical to a single spin measurement, but almost equally effective in satisfying Criterion 5), a bias voltage is applied between the two Ps so that the electron will tunnel from one P to another if a final state is available (the singlet) but not otherwise. Then, again, electrometry can be used to determine whether the P has become charged or not.

3.2. SUPERCONDUCTING QUBITS

We will not review the superconducting proposals in detail, but we will give a brief idea of how the five criteria are to be satisfied and indicate to the reader where he can find more information on these developments. The proposals fall into two broad classes: ones in which qubits involve the charge degree of freedom of the superconductor, and ones where the qubits are embodied by the flux states in a superconducting structure.

The charge models[35, 36] use as their basic structure a pair of small superconducting islands separated by a Josephson junction. The isolation of the islands causes the Cooper-pair number on the islands to be almost a good quantum number; a qubit can be defined as one in which a Cooper pair is resident on either the left or the right island. Changing the voltages on the islands and, possibly, the strength of the effective Josephson coupling, provide, in principle, sufficient control to do one-bit gates, although it is not clear whether the necessary parameters can be controlled with sufficient precision, and whether "1/f" phenomena (random switching of impurities near the device) can be adequately suppressed. Two-bit gate action is obtained by electrostatic coupling between islands[35] or coupling via the quantum states of an LC circuit[36]. This coupling is not formally scalable to large numbers of qubits, but it would be feasible for modest (10) numbers of qubits. Readout would be by single-electron electrometry, which is reasonably well understood. The standard phenomenological model of Josephson circuits suggest that the decoherence rates will be low enough that gate operations can be performed.

The flux models come in a number of varieties[37, 38]; the approach favored by the experimentalists[39] involves a low-Tc (Al or Nb) SQUID circuit which, classically, has two degenerate low energy states which differ slightly in their flux configuration; from the quantum point of view, this is a double-well potential problem with two nearly degenerate ground states, which functions as the qubit. The observation of single-qubit control is then equivalent to the "MQC" phenomenon which has been sought in these structures for many years. Controlled inductive coupling between SQUIDs is proposed for two-qubit gate action. Again, solutions for the five requirements are all proposed; it is possibly worrisome that "extrinsic" effects resulting from the large-scale motion of magnetic flux which might cause decoherence, e.g., coupling to stray spins in the substrate, have apparently not been evaluated.

3.3. OPTICAL

We would finally like to give a very brief mention of another general line of attack that has some promise for solving some of the problems contained in the five criteria: the use of solid-state optical physics. We have not studied these schemes in great detail, but it is clear that they have the prospect of providing some of the best solutions to the problems, for example, of single-quantum measurement. It is also clear that optical expertise is of great value in the characterization of the quantum behavior of spins; all the recent determinations of decoherence times of spins in semiconductors[21] were performed using optical techniques.

So far, many of these proposals are not fully worked out; for the most part, they conceptually follow the quantum-optics proposal involving trapped atoms in a small optical cavity of Pellizzari *et al.*[11]. For example, the proposal of Brun and Wang [40] uses a very familiar object from quantum optics, the whispering-gallery optical modes of a silica microsphere, to couple quantum dots. Other optical cavities with reasonably high Q are available in solid state physics: in the proposal of Imamoglu and coworkers[41], the microdisk optical cavity, a common structure in laser research, is studied;

in this structure the optical cavity is formed by a pair of circular mirrors created by the deposition of multiple layers of different III-V semiconductor materials (typically) with different dielectric constants. the mode volume is typically also occupied by another III-V semiconducting material, but it can also contain a collection of quantum dots. In a manner related to [11] (and also to the more recent proposal [42]), the quantized cavity modes provide a means of turning on and off two-bit interactions between individual spins in the quantum dots (again, the qubit of choice). Single-qubit operations are accomplished by near-resonant two-beam Raman transitions obtained by directing (classical) light of the right frequency at particular quantum dots. The decoherence properties are estimated to be marginal given the current state of the art of these microcavities, but this technology is expected to continue to improve.

This and other proposals are at a concrete enough stage that experiments can begin to probe the ability to satisfy some of the five criteria. Many related approaches are possible, and we only mention the electric-dipole transition qubits proposed by Sherwin *et al.*[43] (see also [44]), the use of transitions in optical hole burning materials[45], and the use of the spectroscopy of excitons in quantum wells in III-V materials[46]. We don't want to predict where any of these approaches will lead us, but they all have the possibility of future success.

4. Quantum Communication with Electrons

The essential resources for quantum communication[47] are EPR (Einstein-Podolsky-Rosen) pairs[48]—pairwise entangled qubits—the members of which are shared between two parties ("Alice and Bob"). These parties are located at different places and their goal for instance is to communicate with each other in an absolutely secure way (which is not possible with classical means only). The prime example of an EPR pair considered here is the singlet state formed by two electron spins. The intrinsic non-locality of these states gives rise to striking phenomena such as violations of Bell inequalities and has a number of possible applications in quantum information such as quantum teleportation, quantum key distribution, and entanglement purification. The non-locality of EPR pairs has been experimentally investigated for photons[49, 50], but not yet for *massive* particles such as electrons, let alone in a solid state environment. This is so because it is difficult to first produce and to then detect entanglement of electrons in a controlled way. In the following we review two scenarios we have recently proposed[51, 52] where the entanglement of electrons (once produced e.g. as described in the previous sections) can be detected in mesoscopic transport and noise measurements. One goal of the following discussion is to show that there exists an interesting connection between the field of quantum communication and the field of transport theory in electronic nanostructures. Another goal is to show that the investigation and concrete tests of fundamental phenomena such as quantum non-locality for electrons are within experimental reach within the not-so-distant future. Such phenomena go beyond the standard single-particle interference effects which have been well studied in mesoscopic systems over the last decade or so. Instead, they involve genuine two-particle effects where, due to strong correlations leading to entanglement, the quantum phases of two identical particles interfere with each other in a constructive or destructive way. This two-particle interference manifests itself in observable Aharonov-Bohm phase oscillations in the electric transport current and in the non-equilibrium current-current correlations.

4.1. PROBING ENTANGLEMENT OF ELECTRONS IN A DOUBLE DOT

We consider a double-dot (DD) system which contains two metallic leads which are in equilibrium with associated reservoirs kept at the chemical potentials $\mu_{1,2}$. Each lead is weakly coupled to *both* dots with tunneling amplitudes Γ, and these leads are probes where the currents $I_{1,2}$ are measured. Note that the DD system is put in parallel in contrast to the standard situation where the coupled dots are put in series (i.e. lead1-dot1-dot2-lead2). The quantum dots contain one (excess) electron each, and are coupled to each other by the tunneling amplitude t, which leads to a level splitting [17, 22] $J = E_{\rm t} - E_{\rm s} \sim 4t^2/U$ in the DD, with U being the single-dot Coulomb repulsion energy, and $E_{\rm s/t}$ are the singlet/triplet energies. We recall that for two electrons in the DD (and for weak magnetic fields) the ground state is given by a spin singlet. For convenience we count the chemical potentials μ_i from $E_{\rm s}$.

The tunneling Hamiltonian [53] reads $H = H_0 + V$, where $H_0 = H_{\rm D} + H_1 + H_2$ with $H_{\rm D}$ describing the DD and $H_{1,2}$ the leads (assumed to be Fermi liquids). The tunneling between leads and dots is described by $V = V_1 + V_2$, where

$$V_n = \Gamma \sum_s \left[D_{n,s}^\dagger c_{n,s} + c_{n,s}^\dagger D_{n,s} \right], \quad D_{n,s} = e^{\pm i\varphi/4} d_{1,s} + e^{\mp i\varphi/4} d_{2,s} \ , \qquad (4)$$

and where $c_{n,s}$ and $d_{n,s}$, $n = 1, 2$, annihilate electrons with spin s in the nth lead and in the nth dot, resp. The Peierls phase φ in the hopping amplitude accounts for an Aharonov-Bohm (AB) or Berry phase (see below) in the presence of a magnetic field. The upper sign belongs to lead 1 and the lower to lead 2. The average current through the DD system is $I = \langle I_2 \rangle$ with $I_n = ie\Gamma \sum_s \left[D_{n,s}^\dagger c_{n,s} - c_{n,s}^\dagger D_{n,s} \right]$.

We consider now the Coulomb blockade (CB) regime where we can neglect double (or higher) occupancy in each dot for all transitions including virtual ones, i.e. we require $\mu_{1,2} < U$. Further we assume that $\mu_{1,2} > J, k_{\rm B}T$ to avoid resonances which might change the DD state. Γ is assumed to be weak (i.e. $J > 2\pi\nu_{\rm t}\Gamma^2$, where $\nu_{\rm t}$ is the tunneling density of states of the leads) so that the state of the DD is not perturbed; this will allow us to retain only the first non-vanishing contribution in Γ to I. In analogy to the single-dot case [54, 55] we refer to the above CB regime as the cotunneling regime. In leading order, the cotunneling current involves the tunneling of one electron from the DD to, say, lead 1 and of a second electron from lead 2 to the DD. However, due to the weak coupling Γ, the DD will have returned to its equilibrium state *before* the next electron passes through it. We focus on the regime, $|\mu_1-\mu_2| > J$, where elastic and inelastic cotunneling occurs, with singlet and triplet contributions being different. In this regime we can neglect the dynamics generated by J compared to the one generated by the bias, and we finally obtain (for $k_BT < \mu_{1,2}$)

$$I = e\pi\nu_{\rm t}^2\Gamma^4 C(\varphi)\frac{\mu_1-\mu_2}{\mu_1\mu_2}\ , \tag{5}$$

$$C(\varphi) = \sum_{s,s'}\left[\langle d_{1s'}^\dagger d_{1s} d_{1s}^\dagger d_{1s'}\rangle + \cos\varphi\langle d_{1s'}^\dagger d_{1s} d_{2s}^\dagger d_{2s'}\rangle\right]\ . \tag{6}$$

Eq. (6) shows that the cotunneling current depends on the properties of the ground state of the DD through the coherence factor $C(\varphi)$ given in (6). The first term in C is the contribution from the topologically trivial tunneling path which runs from lead 1 through, say, dot 1 to lead 2 and the same path back. The second term (phase-coherent part) in C is the ground state amplitude of the exchange of electron 1 with electron 2 via leads 1 and 2 such that a closed loop is formed enclosing an area A. Thus, in the presence of a magnetic field B, an AB phase factor $\varphi = ABe/h$ is acquired.

Next, we evaluate $C(\varphi)$ explicitly in the singlet-triplet basis. Note that only the singlet $|S\rangle$ and the triplet $|T_0\rangle$ are entangled EPR pairs while the remaining triplets $|T_+\rangle = |\uparrow\uparrow\rangle$, and $|T_-\rangle = |\downarrow\downarrow\rangle$ are not (they factorize). Assuming that the DD is in one of these states we obtain

$$C(\varphi) = \begin{cases} 2-\cos\varphi\,, & \text{for the singlet,} \\ 2+\cos\varphi\,, & \text{for all triplets.}\end{cases} \tag{7}$$

Thus, we see that the singlet and the triplets contribute with *opposite sign to the phase-coherent part of the current.* One has to distinguish, however, carefully the entangled from the non-entangled states. The phase-coherent part of the entangled states is a genuine *two-particle* interference effect, while the one of the product states cannot be distinguished from a phase-coherent *single-particle* interference effect. Indeed, this follows from the observation that the phase-coherent part in C factorizes for the product states $T_\pm$ while it does not do so for the entangled states S, T_0. Also, for states such as $|\uparrow\downarrow\rangle$ the coherent part of C vanishes, showing that two different (and fixed) spin states cannot lead to a phase-coherent contribution since we *know* which electron goes in which part of the loop. Finally we note that due to the AB phase the role of the singlet and triplets can be interchanged, which is to say that we can continually transmutate the statistics of the entangled pairs S, T_0 from fermionic to bosonic (like in anyons): the symmetric orbital wave function of the singlet S goes into an antisymmetric one at half a flux quantum, and vice versa for the triplet T_0.

The amplitude of the AB oscillations is a direct measure of the phase coherence of the entanglement, while the period via the enclosed area $A = h/eB_0$ gives a direct measure of the non-locality of the EPR pairs, with B_0 being the field at which $\varphi = 1$. Thus, the measurement of the AB amplitude will provide us with an entanglement dephasing length, which tells us how far we can spatially separate two electrons from each other in a conductor (in the presence of many other electrons, spin-orbit interaction, spin-impurities, etc.) before the entanglement in the total spin state is lost. No doubt, it would be highly desirable to obtain experimental information about this length scale since this will allow us to assess if and under which conditions quantum communication (which makes essential use of separated EPR pairs) will be possible in mesoscopic structures.

The triplets themselves can be further distinguished by applying a directionally inhomogeneous magnetic field (around the loop) producing a Berry phase $\Phi^{\rm B}$ [56], which is positive (negative) for the triplet $m = 1(-1)$, while it vanishes for the EPR pairs S, T_0. Thus, we will eventually see beating in the AB oscillations due to the positive (negative) shift of the AB phase Φ by the Berry phase, $\varphi = \Phi \pm \Phi^{\rm B}$. We finally note that the closed AB-loop can actually be made as large as the dephasing length by using wave guides forming a loop with leads attached to it. Thus, a moderately weak field can be applied to produce the AB oscillations with negligible effect of the orbital state of the DD.

We discuss now the spectral density (noise) of the current cross-correlations, $S(\omega) = \int dt e^{i\omega t} Re\langle \delta I_2(t) \delta I_1(0)\rangle$. Under the same assumptions as before (cotunneling regime), we obtain for the zero-frequency noise its Poissonian value, i.e. $S(0) = -e|I|$. This shows that the Fano factor (noise-to-current ratio) is universal and the current and its cross-correlations contain the same information. For finite frequencies in the regime $|\mu_1 - \mu_2| > J$ and at $T = 0$, we find $S(\omega) = (e\pi\nu_t\Gamma^2)^2 C(\varphi)\,[X_\omega + X^*_{-\omega}]$, where

$$
\begin{aligned}
Im X_\omega &= [\theta(\mu_1 - \omega) - \theta(\mu_2 - \omega)]/2\omega, \qquad (8)\\
Re X_\omega &= \frac{1}{2\pi\omega} sign(\mu_1 - \mu_2 + \omega) \ln|\frac{(\mu_1+\omega)(\mu_2-\omega)}{\mu_1\mu_2}|
\end{aligned}
$$

$$-\frac{1}{2\pi\omega}\left[\theta(\omega-\mu_1)\ln|\frac{\mu_2-\omega}{\mu_2}|+\theta(\omega-\mu_2)\ln|\frac{\mu_1-\omega}{\mu_1}|\right]. \qquad (9)$$

The noise again depends on the phase-coherence factor C with the same properties as discussed before. Here, Re$S(\omega)$ is even in ω, while Im$S(\omega)$ is non–zero (for finite frequencies) and odd, in contrast to single-barrier junctions, where Im$S(\omega)$ vanishes, since $\delta I_1 = -\delta I_2$ for all times. At small bias $\Delta\mu = \mu_1 - \mu_2 \ll \mu = (\mu_1 + \mu_2)/2$, the odd part, Im$S(\omega)$, given in (8) exhibits two narrow peaks at $\omega = \pm\mu$, which lead to slowly decaying oscillations in time, $S_{odd}(t) = \pi\nu_t^2\Gamma^4 C(\varphi)\sin(\Delta\mu t/2)\sin(\mu t)/\mu t$. These oscillations can be interpreted as a temporary charge-imbalance on the DD during an uncertainty time $\sim \mu^{-1}$, which results from the cotunneling of electrons and an associated time shift (induced by a finite ω) between incoming and outgoing currents.

There are a few obvious generalizations to the material presented so far: (1) multi-dot and multiterminal set-ups which implement n-particle entanglement, a prime example being the 3-particle entangled GHZ states *etc.*; (2) variations of the geometries such as the phase-coherent transport from additional "feeding leads" into dots 1 and 2. Such a set-up corresponds topologically to a scattering experiment in which we can arrange for scattering of unentangled electrons (as considered previously in noise studies[57]) but now also of entangled ones. In the latter case we get a non-trivial Fano factor[52] due to antibunching (triplets) and bunching (singlet) effects in the noise[8]: see below. (3) We can replace leads 1 and 2 each by quantum dots which are connected to the double-dot by spin-selective tunneling devices[58] (such spin-filters would allow us to measure spin via charge[17]). Such or similar set-ups would be needed to measure all spin correlations contained in the electronic EPR pairs and thus to test Bell inequalities for electrons in a solid state environment.

4.2. NOISE OF ENTANGLED ELECTRONS: BUNCHING AND ANTIBUNCHING

In this section we discuss a related but alternative scenario in which entanglement of electrons can be measured through a bunching and antibunching behavior in the noise of conductors[8, 52]. The basic idea is rather simple and well known from the scattering theory of two identical particles[59, 60]. In the center-of-mass system the differential scattering cross-section can be expressed in terms of the scattering amplitude $f(\theta)$ and scattering angle θ [60],

$$\sigma(\theta) = |f(\theta) \pm f(\pi-\theta)|^2 = |f(\theta)|^2 + |f(\pi-\theta)|^2 \pm 2Ref^*(\theta)f(\pi-\theta). \quad (10)$$

The first two terms in the second equation are the "classical" contributions which are obtained if the particles were distinguishable, whereas the third term results from the indistinguishability which gives rise to constructive (destructive) *two-particle interference effects*. Here the plus sign applies for spin-1/2 particles in the singlet state (described by a symmetric orbital wave function), while the minus sign applies for their triplet states (described by an antisymmetric orbital wave function). The very same two-particle interference mechanism which is responsible for the enhancement/reduction of the scattering cross section σ near $\theta = \pi/2$ leads to a bunching/antibunching behavior in the statistics[61].

We have previously described in detail how two electron spins can be deterministically entangled by weakly coupling two nearby quantum dots, each of which contains one single (excess) electron[17, 22]. The recently investigated coupling between electrons which are trapped by surface acoustic waves on a semiconductor surface[62] might provide another possibility of producing EPR pairs in a solid-state environment. Generalizing the above two-particle scattering experiment to a mesoscopic system, we have discussed an experimental set-up by which the entanglement of electrons (moving in the presence of a Fermi sea) can be detected in measurements of the current correlations (noise)[8, 52]. For this purpose we employ a beam splitter which has the property that electrons fed into its two incoming leads have a finite amplitude to be interchanged (without mutual interaction) before they leave through the two outgoing leads. In our case, the electrons are entangled before they enter the beam splitter. The quantity of interest is then the noise measured in the outgoing leads of the beam splitter. It is well-known that particles with symmetric wave functions show bunching behavior[57] in the noise, whereas particles with antisymmetric wave functions show antibunching behavior. The latter situation is the one considered recently for electrons in the normal state of mesoscopic transport systems both in theory[63, 64] and in experiments[65, 66]. However, since the noise is produced by the charge degrees of freedom we can expect[8] that in the absence of spin scattering processes the noise is sensitive to the symmetry (singlet or triplet) of only the *orbital part* of the wave function. We have verified this expectation explicitly, by extending the standard scattering matrix approach for transport in mesoscopic systems[63] to a situation with entanglement[52].

The electron current operator in lead α of a multiterminal conductor is

$$I_\alpha(t) = \frac{e}{h\nu} \sum_{E,E',\sigma} \left[a^\dagger_{\alpha\sigma}(E) a_{\alpha\sigma}(E') - b^\dagger_{\alpha\sigma}(E) b_{\alpha\sigma}(E') \right] e^{i(E-E')t/\hbar}, \qquad (11)$$

where $a^\dagger_{\alpha\sigma}(E)$ creates an incoming electron in lead α with spin σ and energy E, and the operators $b_{\alpha\sigma}$ for the outgoing electrons are related to the operators a_α for the incident electrons via $s_{\alpha\beta}$, the (spin- and energy-independent) scattering matrix, $b_{\alpha\sigma}(E) = \sum_\beta s_{\alpha\beta} a_{\beta\sigma}(E)$. Note that since we are dealing with discrete energy states here, we normalize the operators $a_\alpha(E)$ such that $\left\{a_{\alpha\sigma}(E), a_{\beta\sigma'}(E')^\dagger\right\} = \delta_{\sigma\sigma'}\delta_{\alpha\beta}\delta_{E,E'}/\nu$, where the Kronecker symbol $\delta_{E,E'}$ equals 1 if $E = E'$ and 0 otherwise, and ν stands for the density of states in the leads. We also assume that each lead consists of only a single quantum channel; the generalization to leads with several channels is straightforward but is not needed here.

We evaluate the spectral density for the current fluctuations $\delta I_\alpha = I_\alpha - \langle I_\alpha\rangle$ between the leads α and β,

$$S_{\alpha\beta}(\omega) = \lim_{T\to\infty}\frac{h\nu}{T}\int_0^T dt\ e^{i\omega t}\langle\Psi|\delta I_\alpha(t)\delta I_\beta(0)|\Psi\rangle, \tag{12}$$

for the entangled incident state

$$|\Psi\rangle = |\pm\rangle = \frac{1}{\sqrt{2}}\left(a^\dagger_{2\downarrow}(\epsilon_2)a^\dagger_{1\uparrow}(\epsilon_1) \pm a^\dagger_{2\uparrow}(\epsilon_2)a^\dagger_{1\downarrow}(\epsilon_1)\right)|0\rangle. \tag{13}$$

The state $|-\rangle$ is the spin singlet, $|S\rangle$, while $|+\rangle$ denotes one of the spin triplets $|T_{0,\pm}\rangle$; in the following we will present a calculation of the noise for $|+\rangle = |T_0\rangle$, i.e. the triplet with $m_z = 0$. Evaluating the matrix elements we obtain the current correlation between the leads α and β,

$$S_{\alpha\beta}(0) = \frac{e^2}{h\nu}\left[{\sum_{\gamma\delta}}' A^\alpha_{\gamma\delta}A^\beta_{\delta\gamma} \mp \delta_{\epsilon_1,\epsilon_2}\left(A^\alpha_{12}A^\beta_{21} + A^\alpha_{21}A^\beta_{12})\right)\right], \tag{14}$$

where $A^\alpha_{\beta\gamma} = \delta_{\alpha\beta}\delta_{\alpha\gamma} - s^*_{\alpha\beta}s_{\alpha\gamma}$, and $\sum'_{\gamma\delta}$ denotes the sum over $\gamma = 1, 2$ and all $\delta \neq \gamma$, and where again the upper (lower) sign refers to triplets (singlets).

We apply these formulas now to our scattering set-up involving a beam splitter with four attached leads (leads 1 and 2 incoming, leads 3 and 4 outgoing) described by the single-particle scattering matrix elements, $s_{31} = s_{42} = r$, and $s_{41} = s_{32} = t$, where r and t denote the reflection and transmission amplitudes at the beam splitter, respectively. We assume that there is no backscattering, $s_{12} = s_{34} = s_{\alpha\alpha} = 0$. The unitarity of the s-matrix implies $|r|^2 + |t|^2 = 1$. The final result for the noise correlations for the incident state $|\pm\rangle$ is then[67],

$$S_{33}(0) = S_{44}(0) = 2\frac{e^2}{h\nu}T\,(1-T)\,(1 \mp \delta_{\epsilon_1,\epsilon_2})\,, \tag{15}$$

$$S_{34}(0) = 2\frac{e^2}{h\nu}Re\left[r^{*2}t^2\right](1 \mp \delta_{\epsilon_1,\epsilon_2})\,, \tag{16}$$

where $T = |t|^2$ is the probability for transmission through the beam splitter. The calculation for the remaining two triplet states $|+\rangle = |T_\pm\rangle = |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$

yields the same results Eqs. (15) and (16) (upper sign). For the average current in lead α we obtain $|\langle I_\alpha\rangle| = e/h\nu$, with no difference between singlets and triplets. Then, the Fano factor $F = S_{\alpha\alpha}(0)/\,|\langle I_\alpha\rangle|$ takes the form

$$F = 2eT(1-T)\,(1 \mp \delta_{\epsilon_1,\epsilon_2})\,, \tag{17}$$

and correspondingly for the cross correlations. This result implies that if two electrons with the same energies, $\epsilon_1 = \epsilon_2$, in the singlet state $|s\rangle = |-\rangle$ are injected into lead 1 and lead 2, resp., then the zero frequency noise is *enhanced* by a factor of two, $F = 4eT(1-T)$, compared to the shot noise of uncorrelated particles, $F = 2eT(1-T)$. This enhancement of noise is due to *bunching* of electrons in the outgoing leads, caused by the symmetric orbital wavefunction of the spin singlet $|s\rangle$. On the other hand, the triplet states $|+\rangle = |T_{0,\pm}\rangle$ exhibit an *antibunching* effect, leading to a complete suppression of the zero-frequency noise, $S_{\alpha\alpha}(0) = 0$. The noise enhancement for the singlet $|S\rangle$ is a unique signature for entanglement (there exists no unentangled state with the same symmetry), therefore entanglement can be observed by measuring the noise power of a mesoscopic conductor. The triplets can be further distinguished from each other if we can measure the spin of the two electrons in the outgoing leads, or if we insert spin-selective tunneling devices[58] into leads 3,4 which would filter a certain spin polarization.

Note that above results remain unchanged if we consider states $|\pm\rangle$ which are created above a Fermi sea. We have shown elsewhere[8] that the entanglement of two electrons propagating in a Fermi sea gets reduced by the quasiparticle weight z_F (for each lead one factor) due to the presence of interacting electrons. In the metallic regime z_F assumes typically some finite value[68], and thus as long as spin scattering processes are small the above description for non-interacting electrons remains valid.

5. Conclusion

We hope that workers in mesoscopic physics will find this brief survey of recent theoretical developments in quantum computation stimulating. As we continue to learn about quantum computing and quantum communication, we see more and more connections with present-day experimental physics. Quantum computing is not just a mathematical abstraction, it changes our outlook on a variety of fundamental issues in mesoscopics: on the desirability of having long coherence times in mesoscopic structures, on the role of precise time-dependent control of these structures for manipulating the interaction of electron states, on the need to develop high quantum efficiency measurements for spin and other single-quantum properties. Quantum computing and communication clearly have a fascinating role to play in some far-future technologies; we hope that we have illustrated how they can also play a role in the direction of fundamental physics research today.

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68. For instance, in metals such as bulk Cu the quasiparticle weight becomes, within the RPA approximation, $z_F = 0.77$ [69], while for a GaAs 2DEG we find (also within RPA) $z_F = 1 - r_s(1/2 + 1/\pi) = 0.66$ for the GaAs interaction parameter $r_s = 0.61$ (the details of the calculation will be given elsewhere).
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