# Stable Cosmic Vortons

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We present solutions in the gauged $U(1)\times U(1)$ model of Witten describing vortons—spinning flux loops stabilized against contraction by the centrifugal force. Vortons were heuristically described many years ago; however, the corresponding field theory solutions were not obtained and so the stability issue remained open. We construct explicitly a family of stationary vortons characterized by their charge and angular momentum. Most of them are unstable and break in pieces when perturbed. However, thick vortons with a small radius preserve their form in the 3 + 1 nonlinear dynamical evolution. This gives the first ever evidence of stable vortons and impacts several branches of physics where they could potentially exist. These range from cosmology, since vortons could perhaps contribute to dark matter, to QCD and condensed matter physics.

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More than 25 years ago Witten introduced the idea of superconducting cosmic strings in the context of a field theory model that can be viewed as a sector of a grand unification theory (GUT) [1]. The model admits classical solutions describing strings (vortices) whose longitudinal current can attain astronomical values (see Ref. [2] for a review).

Soon after, it was realized that superconducting strings could form loops whose current would produce an angular momentum supporting them against contraction [3]. If stable, such cosmic vortons should be of considerable physical interest, but until recently it was not clear if vortons are stable or not, since the underlying field theory solutions were not known. Various approximations were used to describe vortons, for example, by viewing them as thin and large elastic rings [4]. It was also realized that objects similar to vortons could potentially exist also in other domains, as for example in condensed matter physics [5], or in QCD [6]. Since superconducting strings exist in the Weinberg-Salam theory [7], vortons are potentially possible also there.

The first field theory solutions describing stationary vortons were found in the global limit of Witten's model, when the gauge fields vanish [8]. These vortons have approximately equal radius and thickness, like a Horn torus. Solutions describing thin and large vortons were later found as well; however, when perturbed, thin vortons turn out to be dynamically unstable and break in pieces [9]. Although discouraging, this result is actually quite natural since thin vortons can be locally approximated by straight strings, while the latter are known to become unstable for large currents [2].

However, a more close inspection reveals that unstable modes of superconducting strings have a nonzero minimal wavelength [10], as in the case of the Plateau-Rayleigh instability of a water jet [11]. Therefore, imposing periodic boundary conditions with a short enough period should remove all instabilities. As a result, thick vortons made of short string pieces have chances to be stable.

In this Letter we present for the first time stationary vorton solutions in the gauged Witten model, and our vortons are thick. To study their stability, we simulate their full 3 + 1 nonlinear dynamics in the limit of vanishing gauge couplings. We find that most of them are unstable; however, thick vortons with a large charge and the smallest possible radius are stable. By continuity, it follows that vortons with nonzero but small gauge couplings should be stable as well.

We therefore present evidence of stable vortons, whose features turn out to be quite different from those predicted by the effective theories. This can impact several branches of physics where vortons could potentially exist.

*The model of Witten.*—This is a theory of two Abelian vectors $A_\mu^{(a)}$ interacting with two complex scalars $\Phi_a$ ($a = 1, 2$) with the Lagrangian

$$\mathcal{L} = -\frac{1}{4}\sum_a F_{\mu\nu}^{(a)}F^{(a)\mu\nu} + \sum_a (D_\mu\Phi_a)^* D^\mu\Phi_a - V. \quad (1)$$

Here the gauge field strengths are $F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\nu^{(a)}$, the gauge covariant derivatives $D_\mu\Phi_a = (\partial_\mu + ig_a A_\mu^{(a)})\Phi_a$ with gauge couplings $g_a$, and the potential is

$$V = \sum_a \frac{\lambda_a}{4}(|\Phi_a|^2 - \eta_a^2)^2 + \gamma|\Phi_1|^2|\Phi_2|^2 - \frac{\lambda_2\eta_2^4}{4}, \quad (2)$$

where $\eta_1 = 1$. If $4\gamma^2 > \lambda_1\lambda_2$ and $2\gamma > \lambda_2\eta_2^2$ then the global minimum of the potential (vacuum) is achieved for $|\Phi_1| = 1$ and $\Phi_2 = 0$. Fields $A_\mu^{(1)}$, $\Phi_1$, $\Phi_2$ are massive with masses, respectively, $m_A^2 = 2g_1^2$, $m_1^2 = \lambda_1$, $m_2^2 = \gamma - \frac{1}{2}\lambda_2\eta_2^2$, whereas $A_\mu^{(2)}$ is massless and can be identified with an electromagnetic field. The theory has a local $\mathrm{U}(1) \times \mathrm{U}(1)$ invariance and two Noether currents $j_a^\mu = 2\mathrm{Re}(i\Phi_a^* D^\mu \Phi_a)$ with two conserved charges $\int j_a^t d^3x$. The Euler-Lagrange equations are

$$\partial_\mu F^{(a)\mu\nu} = g_a j_a^\nu, \qquad D_\mu D^\mu \Phi_a + \frac{\partial V}{\partial |\Phi_a|^2}\Phi_a = 0. \quad (3)$$

Assuming cylindrical coordinates $x^\mu = (t, \rho, z, \varphi)$, we make the ansatz for stationary, axially symmetric fields

$$\Phi_1 = X_1 + iY_1, \quad \Phi_2 = (X_2 + iY_2)\exp\{i(\omega t + m\varphi)\}, \quad (4)$$

where $X_a$, $Y_a$ as well as $A_\mu^{(a)}$ depend on $\rho$, $z$, and we impose the gauge condition $A_\rho^{(a)} = 0$. Here $m$ is an integer winding number and $\omega$ is a frequency. The fields should be globally regular and the energy should be finite, which requires that at infinity $X_1 \to 1$ while all other amplitudes approach zero. At the symmetry axis $\rho = 0$, the amplitudes $X_2$, $Y_2$, $A_\varphi^{(a)}$ vanish, while for the other amplitudes the normal derivative $\partial/\partial\rho$ vanishes. Under the reflection $z \to -z$ the amplitudes $Y_a$ are odd whereas all the others are even.

The choice of the ansatz implies that the first Noether charge vanishes, while the second one is

$$Q = 2\int d^3x (X_2^2 + Y_2^2)(\omega - g_2 A_t^{(2)}). \quad (5)$$

The energy is $E = \int T_t^t d^3x$ and the angular momentum

$$J = \int T_\varphi^t d^3x = mQ, \quad (6)$$

where the energy-momentum tensor is obtained by varying the metric tensor $T_\nu^\mu = 2g^{\mu\sigma}\partial\mathcal{L}/\partial g^{\sigma\nu} - \delta_\nu^\mu \mathcal{L}$. In the above formulas all fields and coordinates are dimensionless. If $\boldsymbol{\eta}$ is the energy scale, then the dimensionful (boldfaced) quantities are $\boldsymbol{\Phi}_a = \boldsymbol{\eta}\Phi_a$, $\mathbf{A}_\mu^{(a)} = \boldsymbol{\eta} A_\mu^{(a)}$, $x^\mu = \mathbf{x}^\mu \boldsymbol{\eta}$, $\mathbf{E} = \boldsymbol{\eta} E$; hence, $\boldsymbol{\eta}$ is the asymptotic value of $\boldsymbol{\Phi}_1$.

*Stationary vortons.*—Inserting the ansatz (4) to the field equations (3) gives, after separating the $t$ and $\varphi$ variables, an elliptic system of ten nonlinear partial differential equations for the ten functions of $\rho$, $z$. We solve these equations with two different numerical methods: using the elliptic partial differential equation solver FIDISOL based on the Newton-Raphson procedure [12], and also minimizing the energy within a finite element approach provided by the FREEFEM++ library [13].

We look for solutions with a toroidal structure and nontrivial phase windings along both torus generators. Apart from the azimuthal winding number $m$, there is a second integer $n$ determining the winding of the phase of $\Phi_1$ around the boundary of the $(\rho, z)$ half plane. If $n \neq 0$ then $\Phi_1$ vanishes at a point $(\rho_0, 0)$ corresponding to the center of the closed vortex forming the vorton, and the phase of $\Phi_1$ winds around this point. Prescribing nonzero values of $n$, $m$, and $Q$ (see the Supplemental Material [14] for details), the fields cannot unwind to vacuum, and the iterative numerical procedure converges to a smooth limiting configuration with a finite radius $\rho_0$. We have constructed in this way vortons for $n = 1, 2$ and $m = 1, \ldots, 12$, and also solutions similar to $Q$ balls [15] for $n = 0$, $m = 0, 1, \ldots$ (see the Supplemental Material [14]).

The vorton can be visualized as a toroidal tube confining a magnetic flux of $\vec{B}^{(1)} = \vec{\nabla} \times \vec{A}^{(1)}$, since $\Phi_1 \approx 0$ inside the tube and thus the first U(1) is restored. $\Phi_2$ is nonzero inside the tube, giving rise to a charged condensate and to a persistent electric current along the tube. The current creates a momentum along the azimuthal direction, which gives rise to an angular momentum along the $z$ direction. Outside the vorton tube the massive fields $A_\mu^{(1)}$, $\Phi_1$, $\Phi_2$ rapidly approach their vacuum values and there remains only the long-range massless $A_\mu^{(2)}$ generated by the electric current confined inside the vorton tube. At large $r = \sqrt{\rho^2 + z^2}$ one has $A_t^{(2)} = Q/(4\pi r) + \cdots$ and $A_\varphi^{(2)} = \mu \sin\theta/r^2 + \cdots$; therefore, from far away the vorton looks like a superposition of an electric charge $Q$ with a magnetic dipole $\mu$.

Figure 1 shows the 3D solution profiles for an $m = 1$ vorton. One can see that the vorton tube is very thick and compact. The vortex magnetic field $\vec{B}^{(1)} = \vec{\nabla} \times \vec{A}^{(1)}$ and the electric current $\vec{j}_2$ are tangent to the azimuthal lines. The electric field $\vec{E}^{(2)} = -\vec{\nabla} A_t^{(2)}$ is mostly oriented along the radial direction and supports a nonzero flux at infinity, $Q = \oint d\vec{E}^{(2)} \cdot d\vec{S} = g_2 Q$. The massless magnetic field $\vec{B}^{(2)} = \vec{\nabla} \times \vec{A}^{(2)}$ at large $r$ is of magnetic dipole type.

Vortons can be labeled by their charge $Q$ and the integer spin $m = J/Q$ (assuming that $n = 1$). The vorton radius $\rho_0$ is not very sensitive to the value of $Q$ but increases rapidly with $m$, so that for large $m$ vortons are thin and large, with the radius much larger than the thickness. On the other hand, increasing $Q$ increases the thickness of the vorton tube, so that for large $Q$ vortons are thick and look almost spherical.

The frequency $\omega$ can be used instead of $Q$ to characterize the solutions, which exist only within a finite frequency range $\omega_- < \omega < \omega_+$. Both $E$ and $Q$ diverge for $\omega \to \omega_\pm$ and have a minimum in between, as shown in Fig. 2. Vortons can be viewed as boson condensates, which is why their charge cannot be too small, since the boson condensation is not energetically favored for small quantities of the field quanta.

For $g_a = 0$ the gauge fields vanish and the vortons are global, made of the scalars $\Phi_a$ alone [8]. For $g_a \neq 0$ the gauge fields are excited and increase the total energy and charge, as shown in Fig. 2. Solutions do not exist for arbitrary values of $\lambda_a$, $\eta_2$, $\gamma$, $g_a$ but only for some regions in the parameter space. For example, fixing all parameters and also $\omega$ and varying $g_1 = g_2$, the solutions exist

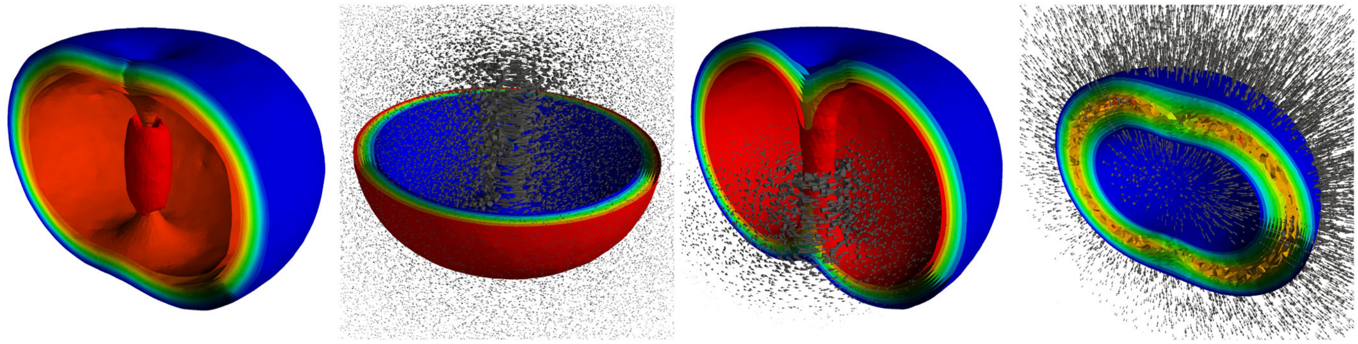

FIG. 1 (color online). Profiles of the stationary vorton solution for $Q = 1500$ and $n = 1$, $m = 1$ for the parameter values $\lambda_1 = 41.1$, $(\lambda_2, \eta_2) = (30, 1)$, $\gamma = 20$, and $g_1 = g_2 = 0.01$. The first panel displays constant energy surfaces. The second panel shows surfaces of constant $|\Phi_1|^2$ and the magnetic field $\vec{B}^{(1)}$ (cones). One has $\vec{E}^{(1)} = 0$. The third panel shows $|\Phi_2|^2$ isosurfaces and the electromagnetic current $\vec{j}_2$ (cones). The last panel shows the electric field $\vec{E}^{(2)}$ (cones), while the isosurfaces show its magnitude. The gray (red online) corresponds to large values, the dark gray (blue online) to small values, and the light gray (yellow online) to intermediate values.

only within a finite range of gauge couplings, as is seen in Fig. 2.

*Dynamical vortons.*—To analyze the vorton stability, we simulate their nonlinear $3 + 1$ temporal dynamics. In doing this, we consider only the global vortons, since simulating dynamics of the gauge fields would require too much computer power. However, we expect the results obtained in the global case to apply to the fully gauged vortons as well, at least for small enough gauge couplings $g_a$. Indeed, for $g_a = 0$ the vorton is made of scalars $\Phi_a$. For small nonzero $g_a$ their global currents give rise to the $O(g_a)$ source in the gauge field equations; hence, $A_\mu^{(a)} = O(g_a)$. The backreaction of the gauge fields on the scalars is $O(g_a^2)$ and can be neglected as compared to the reaction of the scalars on themselves. For stationary vortons this is confirmed by the numerics, as is seen in Fig. 2. Therefore, one can expect the temporal dynamics of vortons with small $g_a$ to be dominated by the scalars only; hence it can be approximated by the global vorton dynamics.

Vortons with large $m$ develop pinching deformations breaking them in pieces [9]. However, for small $m$, vortons could be stable, since they are compact and thick and have no room for the instability to settle in. To verify this, we consider a hyperbolic evolution scheme based on an implicit $\beta$-Newmark finite difference approximation (see the Supplemental Material [14] for details). The initial configuration is a stationary, axially symmetric vorton. It becomes automatically perturbed by the space discretization when adapted from 2D to the 3D mesh, which triggers a nontrivial temporal evolution. The natural time scale is set by the value of $\omega$ of the underlying vorton solution, which is of order one. We integrate with the time step $\Delta t = 0.1$ and find that the $m \geq 3$ vortons very quickly become strongly deformed and then break in pieces. The time they take to break decreases rapidly as $m$ grows (see Fig. 3 and the Supplemental Material [14] for the videos). The products of the vorton decay are typically two or three out-spiraling fragments of spherical topology.

We therefore conclude that thin and large vortons are unstable, thus confirming the result of Ref. [9]. One should say that a different conclusion was previously made in Ref. [16], where thin vortons were found to be stable. Since neither our analysis nor that of Ref. [9] confirms this, it is possible that the conclusion of Ref. [16] is an artifact of modifying the scalar potential made in that work in order to improve the stability behavior.

We finally turn to vortons with $m = 1, 2$ and choose a large value $Q$, in which case the vortons are compact and thick. For $m = 2$ we cannot make a definite conclusion, since the vortons do not actually break but sometimes become strongly deformed. However, nothing at all happens to the $m = 1$ vortons. As the time increases, they only move slowly in the box, sometimes reflecting from the boundary, without changing shape. We integrated up to $t \sim 10^3$ (which requires weeks of runtime) without

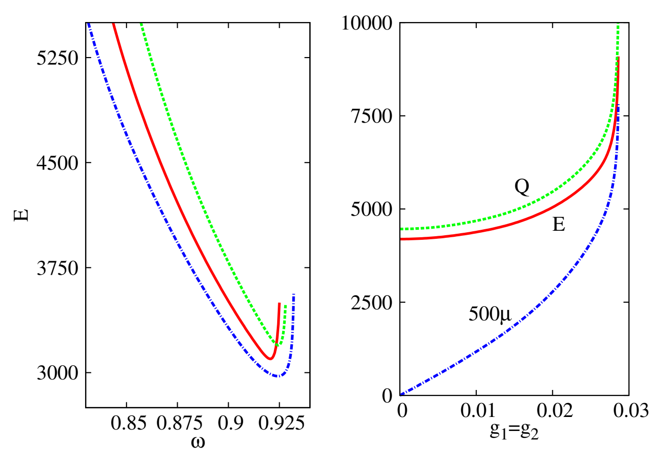


FIG. 2 (color online). Left panel shows $E$ against $\omega$ for the values of gauge couplings $(g_1, g_2) = (0.008, 0.021)$ (dashed), $(0.012, 0.012)$ (solid), $(0.008, 0.027)$ (dotted). Right panel shows $E$, $Q$, $\mu$ against $g_1 = g_2$ for a fixed $\omega = 0.804$. In both cases $\lambda_1 = 41.1$, $\lambda_2 = 40$, $\gamma = 22.3$, $\eta_2 = 1$, $n = 1$, $m = 2$.

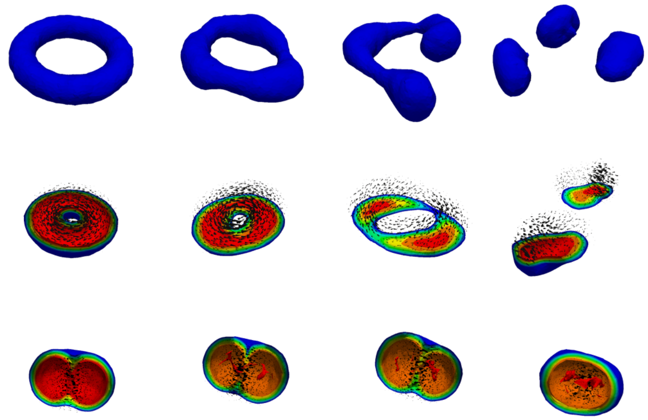

FIG. 3 (color online). Snapshots of the vorton time evolution. The first line shows a constant $|\Phi_2|^2$ surface for the $m = 6$ solution for $t = 0, 17, 21, 28$. The second and third lines show the constant $|\Phi_1|^2$ surfaces and the electric current, respectively, for the $m = 3$ vorton for $t = 0, 31, 28, 47$ and for the $m = 1$ vorton for $t = 0, 104, 153, 239$. The third line shows the stable solution for $Q = 6000$ and for the same values of $\lambda_a, \eta_2, \gamma$ as in Fig. 2.

noticing any change in their behavior. We also checked that increasing the size of the box does not change anything, so that one cannot say that the boundary has a stabilizing effect. We therefore conclude that vortons with the lowest spin and a large charge are dynamically stable (see the Supplemental Material [14] for more discussion). Intuitively, this is because they are so thick that they are hard to pinch.

Interestingly, a very similar conclusion was made for the spinning light bullets, which share many properties with vortons [17]. These are nonrelativistic solutions for a complex scalar field with a $t$, $\varphi$-dependent phase (as for $\Phi_2$), and they also have toroidal profiles. Their $E(\omega)$ dependence is similar to that shown in Fig. 2. It was found that the $m = 1$ solutions with a large charge and $\omega$ close to $\omega_-$ preserve their shape in the $3 + 1$ dynamical evolution [17]. Exactly the same statement applies for our relativistic vortons.

In summary, for the first time since the vortons were heuristically described almost 25 year ago [3], we present the underlying stable solutions within the $U(1) \times U(1)$ gauge field theory of Witten [1]. We can now make some estimates. Assuming the original motivation of Witten, the energy scale should be of the GUT magnitude $\boldsymbol{\eta} \sim 10^{14}$ GeV. Using the value $E \sim 5 \times 10^3$ for estimates, it follows that vortons are extremely heavy, with $\mathbf{E} \sim 5 \times 10^{17}$ GeV, which is not far from the Planck energy. On the other hand, their minimal Noether charge $Q \sim 5 \times 10^3$ is actually not so large as compared to the average particle density in the hot early universe. Therefore, vortons could be abundantly created due to charge fluctuations in the course of a phase transition via the Kibble mechanism [18], if only GUTs indeed applied in the past. Being classically stable, vortons could disintegrate via a quantum tunneling towards the $\rho_0 = 0$ state, but this process should be exponentially suppressed, and in fact quantum fluctuations can also have a stabilizing effect by preventing the collapse to zero size [19]. Vortons could probably evaporate via interactions with GUT fermions [20], but this process should stop after the GUT epoch. Therefore, it is not inconceivable that some relic vortons could still be around and contribute to dark matter.

Let us consider $g_a = 0$ vortons. For stationary fields (4), Eqs. (3) for $\Phi_a$ can formally be interpreted [8] as the nonrelativistic Gross-Pitaevskii equation for a two-component Bose-Einstein condensate (BEC). This can describe ultracold atomic gazes with two hyperfine states, such as $^{87}$Rb [21]. Scalars $\Phi_a$ then correspond to the two BEC order parameters, one of which creates a vortex while the other one condenses in the vortex core. Our solutions therefore describe stationary vortons made of loops of such vortices, whose potential existence has been much discussed [5]. In fact, vortex rings in two-component BECs have been observed experimentally [22], although it is not completely clear if they support an angular momentum [23].

$\Phi_1$ and $\Phi_2$ can also be interpreted [24], respectively, as the $d$-wave superconducting and antiferromagnetic order parameters in the SO(5) model of high $T_c$ superconductivity [25]. This model admits $d$-wave superconducting vortices with an antiferromagnetic core [26], while our solutions describe loops made of these vortices. Such vorton quasiparticles could be important for the superconducting phase transition in this model [24].

Equally, scalars $\Phi_a$ can be viewed as describing a condensate of ($K^+$, $K^0$) mesons in QCD [27]; hence, our solutions describe the $K$ vortons, whose existence was conjectured in Ref. [6]. Setting the scale to be $\boldsymbol{\eta} \sim 200$ MeV gives for their energy $\mathbf{E} \sim 1$ TeV. Such objects could probably exist in dense QCD matter, as for example in neutron stars [28], which may affect their electromagnetic and neutrino transport properties.

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